Uncertainty in Shear Wave Velocity Based on Standard Penetration Test by Using Error Least Square Model

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Abstract

Shear wave velocity (Vs) is a basic engineering soil property implemented in evaluating the soil shear modulus. Due to a few limitations, sometimes it is preferable to determine Vs indirectly by in situ tests, such as standard penetration test (SPT). However, inaccuracies in measurement or estimation of the influencing parameters have always been a major concern, and thus various statistical approaches have been proposed to subdue the effect of such inaccuracies in predictions of future events. In this article, an innovative approach based on robust optimization has been utilized to enumerate the effect of such uncertainties. In order to assess the merits of the proposed approach a database containing 326 data points of case histories from Adapazari, Turkey were gathered from renowned references. The identification technique used in this article is based on

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the robust counterpart of the least square problem which is a second order cone problem and is efficiently solved by interior point method. A definition of uncertainty based on frobenius norm of the data is introduced and examined against correlation coefficient of various correlation parameters and optimum values are determined. Finally the results of new correlation are compared with those utilizing a commonly used statistical method and the advantages and possibilities of the proposed correlation over the conventional method are highlighted.

KeyWords: Shear wave velocity; Standard penetration test; Least squares; Uncertainties; Robust optimization; Second order cone.

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Introduction

Shear wave velocity is a basic engineering soil property for earthquake site response analysis, which is directly related to shear modulus at small shear strain level. Therefore, Vs is one of the indirect methods to evaluate soil modulus. Because of difficulties in soil sampling, the high cost for obtaining the high-quality undisturbed samples which represent confining stress conditions, in situ investigation such as down hole is preferred to laboratory tests. It is preferable to determine Vs directly by in situ tests, such as seismic measurements. Using surface wave velocity measuring techniques, a shear wave velocity profile can be established without boring and

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penetration [1]. The nondestructive, non-intrusive features make Vs - based approach a potentially attractive alternative for characterizing liquefaction resistance in sandy soils [2]. However, this is not always feasible, due to space constraints and, especially in urban areas, high noise levels associated with these tests. Therefore, it is necessary to determine Vs through indirect methods such as SPT and CPT which are commonly used for usual geotechnical site investigations.

In geotechnical engineering, many design parameters of soil are associated with the standard penetration test (SPT). Standard penetration Test blow counts, N_{SPT} is significant in site investigation, along with other geotechnical parameters such as Vs. There is no theoretical relationship between destructive (e.g. SPT and geotechnical soil parameters) and non-destructive methods (e.g. seismic methods); hence, their association, and evaluation of geotechnical properties, requires empirical correlations, statistical analysis and system identification techniques. The interdependency of factors involved in such problems prevents the use of regression analysis methods such as least square. Inaccuracies in measurement or estimation of the influencing parameters and least square cannot predict Vs correctly. Therefore a new approach "Error Least Square Model" proposed to quantification of the effect of uncertainties on evaluation of correlation parameters in this study. This model is the robust counterpart of the least square model, which is a second order cone program (SOCP) in which, possible uncertainties can be reasonably adjusted [3].

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Background to previously proposed correlations

Many research works can be found in the literature regarding application of N_{SPT} for geotechnical characterization. Imai and Yoshimura [4] studied the relationship between seismic velocities and some index properties over 192 specimens and developed empirical relationships for all soils. Sykora and Stokoe [5] pointed out the geological age. Jafari et al., [6] present detailed historical review on the statistical correlation between N_{SPT} versus Vs in fine grained soils. Some researchers have proposed correlations between N_{SPT} and Vs for different soils, such as sand, silt and clay. Hasancebi and Ulusay [7] studied statistical correlations in sand and clay soil types except for gravels. Ulugergerli and Uyanık [8] investigated statistical correlations using 327 samples and defined the empirical relationship as upper and lower bounds instead of a single average curve for estimating seismic velocities and relative density. Dikmen [9] investigated uncorrected SPT data and presented a correlation for all type of soils. Others have developed correlations which included stress-corrected Vs, energy-corrected N_{SPT} (e.g. Pitilakis et al., [10], Kiku et al., [11]), energy- and stress-corrected N_{SPT}, depth (e.g. Tamura and Yamazaki [12]) and fine content (e.g., Ohta and Goto [13]). The shear wave velocity can also provide estimation of effective stress (σ'_v) that Mayne et al., [14] suggested in clay soils type. Mayne [15] presents a relationship between the total unit weight (γ) in terms of Vs and depth (Z) for

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saturated soils. However, almost all of the studies mentioned above focused on the relationships between uncorrected N_{SPT} and Vs for all soils as well as sand and clay-type soils. Some of these empirical relations have shown in Table.1

Table1. Some of the existing correlations between uncorrected N_{SPT} and Vs.

Ref.	Proposed Relation for all soils
Imai and Yoshimura (1975)	$Vs = 89.9\ N^{0.341}$
Ohta and Goto (1978)	$Vs = 85.35\ N^{0.348}$
Sykora and Stokoe (1983)	$Vs = 100.5\ N^{0.29}$
Jafari et al (1997)	$Vs = 22\ N^{0.85}$
Kiku et al (2001)	$Vs = 68.3\ N^{0.292}$
Jafari et al (2002)	$Vs = 27\ N^{0.73}$ (clay type)
Hasancebi and Ulusay (2007)	$Vs = 90\ N^{0.309}$
Ulugergerli and Uyanık (2007)	a-$V_SU = 23.291Ln(N)+ 405.61$ b-$V_SL = 52.9\ e^{-0.011N}$
Dikmen (2009)	$Vs = 58N^{0.39}$

a Upper bound **b Lower bound**

Overview of database and case histories

The Kocaeli earthquake by Moment Magnitude of 7.4 had occurred in 1999 and the epicenter was located near the city of Izmit. The fault rupture was physically visible throughout most of the seismically impacted area from Karamürsel to Akayazi. The cause of the earthquake was a multiple rupture process in 140km long western part of 1200 km. In the vicinity of Adapazari peak ground accelerations were recorded at approximately 0.4 g. In the region, as many as 70% of the buildings were subjected to large ground settlements, liquefaction, or subsidence and sea water inundation [16]. As illustrated in Fig.1, the southern shores of Izmit Bay are covered by Holocene deposits except a relatively small area, which was classified geologically as Backpack formation of Companian-Maastrihtien age consisting of marl, mudstone, conglomerate and sandstone. From a sediment logical point of view, the southern shores of Izmit Bay are covered principally by fine-grained sandy deposits which get finer (siltier and more clayey) towards the north into the depths of Izmit Bay. A total of 135 CPT profiles (of which 19 were seismic CPTs) and 46 soil borings with multiple SPT (often at 0.8m spacing) were completed in the city of Adapazari. As shown in Fig.2 soil profiling in the Police station site located on the east shore of Izmit Bay, in the town of Golcuk, the soil liquefaction suspected is considerable [17].

(http://peer.berkeley.edu/turkey/adapazari/)

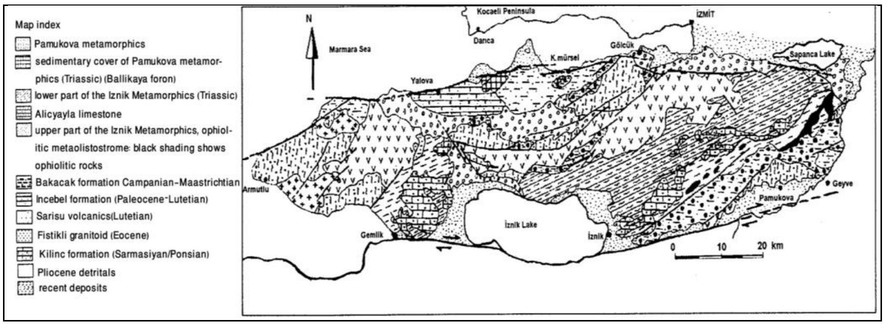

Fig.2. Simplified geological map of Armutlu peninsula (after Goncuoglu [18])

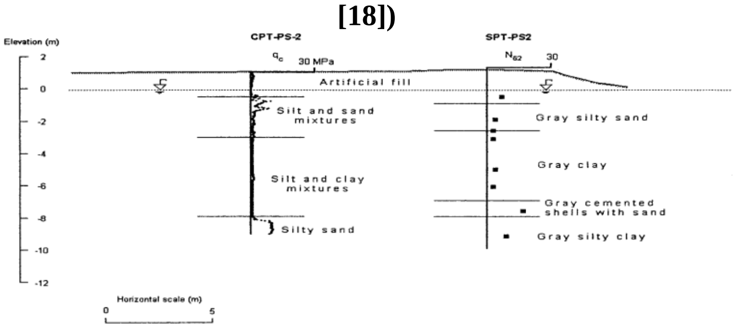

Fig.2. Soil profiling in the Police station site located on the east shore of Izmit Bay, in the town of Golcuk [17]

Table3. A sample of the database [16]

Z (m)	$(N_1)_{60}$	FC(%)	Vs(m/s)
3.3	6	83	170
17.8	31	80	294
2	6	56	110
2.7	12	53	110
18.2	18	5	262
1.4	4	99	100
9	6	98	200
16.2	13	74	172
6.8	23	14	151
2.6	6	92	253
7.9	40	11	150
14.8	5	98	172
13.2	30	20	179
2.5	13	65	105
2.8	4	99	121
6.5	8	99	95
9	48	5	250
2.6	4	99	85
7.7	39	11	150
17.8	16	12	243
4.1	25	71	306
3.4	4	78	150

The variables considered for the proposed correlations

The dataset, explained in [16] consists of 326 case records for Adapazari, Turkey. The database, a random sample selection given in Table 3, covers a wide spectrum of soils and seismic parameters, including soil layer depth (Z), corrected SPT blow number ($N_{1\ 60}$), FC or Fines Content (% ≤ 75µm) and shear wave velocity (Vs). The objective of selecting above parameter is to limit the disadvantages of SPT utilization in clay soil type and the effect of overburden stress in Vs evaluation. Further details regarding the measurement and interpretation of the foregoing parameters are available in [16]. Fig.3

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illustrates the distribution of the variable characteristics for all case studies.

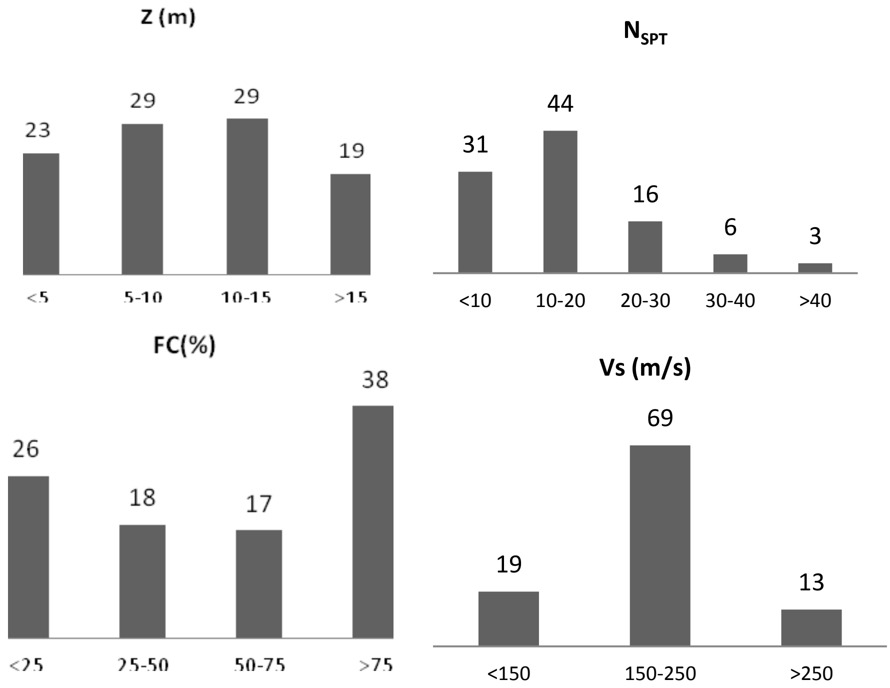

Fig.3. Distribution of the variable characteristics for all case histories

Development of the new correlation

In this paper according to the previous studies, there correlations proposed as follow:

Correlation 1)

$$Vs = a_1 N^{a_2} \tag{1}$$

Correlation 2)

$$Vs = a_1 Z^{a_2} N^{a_3} \tag{2}$$

Correlation 3)

$$Vs = a_1 Fc^{a_2} Z^{a_3} N^{a_4} \tag{3}$$

These correlations can be definite in the form of Ax=b in system of log, where $A_{m\times n,}$ (m>n) and $b_{n\times 1}$. As the collected database includes

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possible uncertainties, the classical method for solving least squares problems, i.e.

$$\min_{x \in R^n} \| Ax - b \| \tag{4}$$

will not provide proper results. In the next section the robust counter will be discussed

Robust optimization model

In mathematical optimization models, it is commonly assumed that the data inputs are precise and the influence of parameter uncertainties on the feasibility of the models are ignored. It is therefore conceivable that as the data differ from the assumed nominal values, the generated optimal solution may violate critical constraints and perform poorly from an objective function point of view. These observations motivate the need for methodologies in mathematical optimization models that account for solutions immune to data uncertainty [3].

Robust optimization addresses the issue of data uncertainties from the perspective of computational tractability. In the past decade, there were considerable developments in the theory of robust convex optimization. However, under the robust framework found in the literature, the robust models generally lead to an increase in computational complexity over the nominal problem, which is an issue when solving large problems [19].

In the sequel, a robust model for the least squares method which we considered as the correlation (Table 2) is presented. Suppose that the level of uncertainty of a database is known and equal to ρ. Then the

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robust model which considers this level of uncertainty in the database, minimizes the worst case residual, i.e.

$$min_x max_{\|[E,r]\|_F \le \rho} \|(A+E)x-(b+r)\| \tag{5}$$

where E and r are uncertainties in A and b, respectively and the matrix norm is the Frobenius norm, which for a given matrix A is defined as $\|A\|_F = (\sum_i^n \sum_j^m A_{ij}^2)^{\frac{1}{2}}$. Obviously problem (Eq.4) cannot be solved using classical optimization algorithms. However, it can be written in the following Second Order Conic Programming (SOCP) form [20]:

$$\begin{cases} \min(t+\rho s) \\ \|Ax-b\| \le t \\ \sqrt{1+\|x\|^2} < s \end{cases} \tag{6}$$

Eq. 6 can be solved using efficient software like SeDuMi [20], which is an interior point based software for solving SOCP and Semi definite Optimization. It may be noted from an unconstrained least squares problem, a SOCP is developed that is harder to solve, but is more conservative.

Thus problem (Eq. 6) is rewritten in the dual form of SeDuMi's input format, namely

$$\begin{cases} max b^T y \\ c - A^t y \in K \end{cases} \tag{7}$$

where

$$c = \begin{pmatrix} 0 \\ -b \\ 0 \\ 1 \\ 0_{n\times1} \end{pmatrix}, A^t = \begin{pmatrix} -1 & 0 & 0_{1\times n} \\ 0_{m\times1} & 0_{m\times1} & -A \\ 0 & -1 & 0_{1\times n} \\ 0 & 0 & 0_{1\times n} \\ 0_{n\times1} & 0_{n\times1} & -I_{n\times n} \end{pmatrix}, b = \begin{pmatrix} -1 \\ -\rho \\ 0_{n\times1} \end{pmatrix}, Y = \begin{pmatrix} t \\ s \\ x \end{pmatrix} \tag{8}$$

$$K = Q_{m+1} \times Q_{n+2} \tag{9}$$

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Where Q_k denotes the second order cone in R^k and is defined as follows:

$$Q_k = \{x \in R^k | \|\bar{x}\| \leq x_1\}, \bar{x} = (x_1, \dots, x_{k-1})^T \quad (10)$$

By this definition, one now can easily see that the first constraint in (Eq. 6) belongs to Q_{m+1} and the second one belongs to Q_{m+2}. These are denoted in SeDuMi’s format by the product of these two second order cones, namely K.

Then SeDuMi is called by the following command for four different values of uncertainty parameter ρ:

[x,y]=sedumi(At,b,c,K) (11)

where At denotes the matrix A^t in (Eq. 8), b, c also are taken from (Eq. 8) and K also is given by (Eq. 9). Moreover, x and y denote the solutions of (Eq. 7) and its dual problem.

In order to consider uncertainty a new parameter was introduced as belw:

$$Uncertainity = \frac{\rho}{\|Data\|_F} \times 100 \quad (12)$$

Where $\|Data\|_F$ is the Frobenius norm of data matrices as defined before. The results of constant factors ai through the uncertainties are summarized in Fig.4.

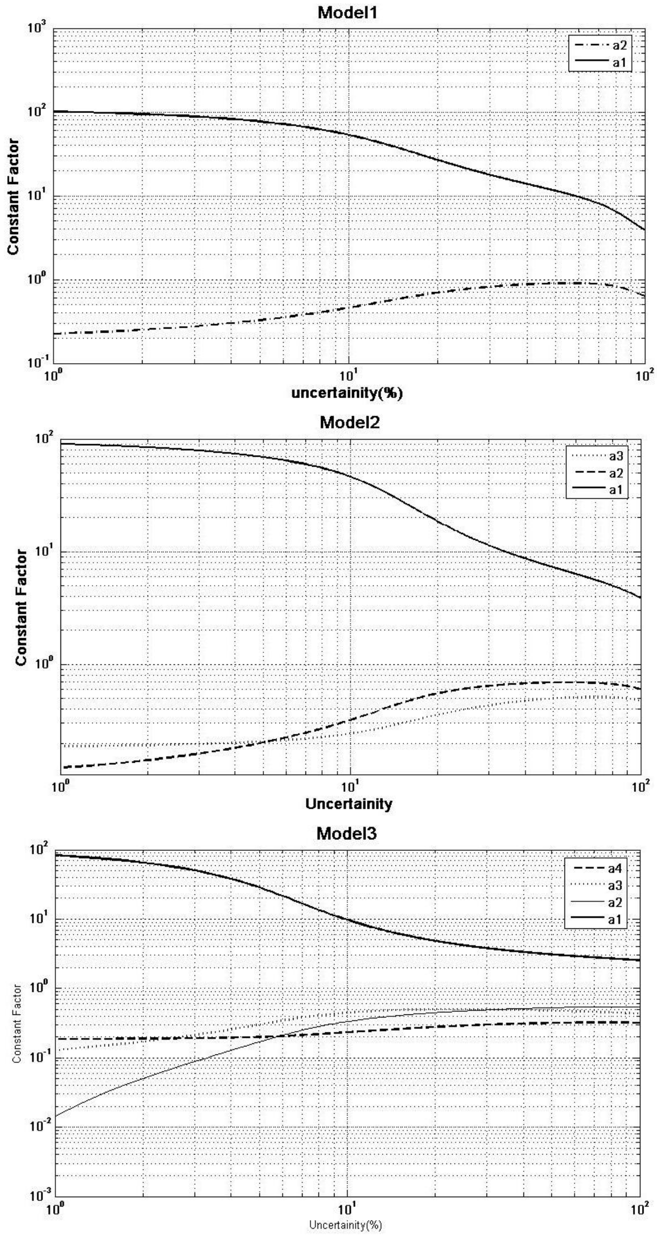

Fig.4. The variation of constant coefficients of different correlations versus uncertainties

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In order to determine the accuracy of correlation statistical value R^2 as absolute fraction of variance, can be used defined as follows [21]:

$$R^2 = 1 - \left[\frac{\sum_{i=0}^{M}\left(Y_{i(Correlation)} - Y_{i(Actual)}\right)^2}{\sum_{i=1}^{M}\left(Y_{i(Actual)}\right)^2}\right]$$

The results are summarized in Fig.5. Therefore correlation three, by considering fines content the R^2 has a little difference.

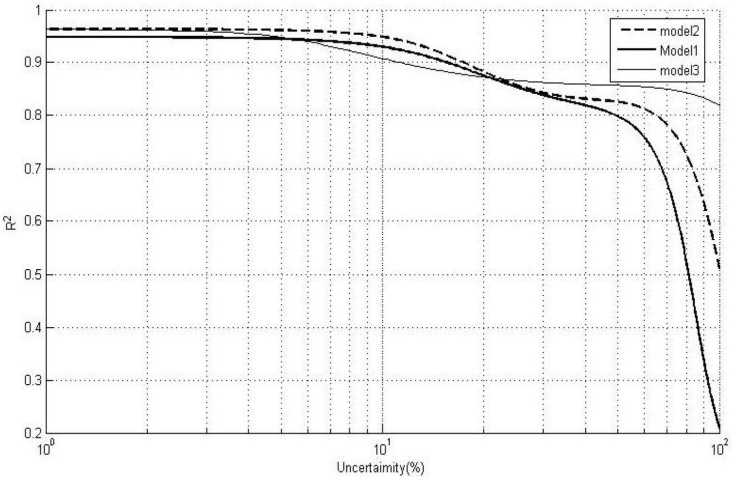

Fig.5. The variation of regression coefficient (R^2) of different correlations versus uncertainties

Comparison of correlations

The accuracy of the proposed correlation, in predicting shear wave velocity, is compared with recently correlations presented by Kiku et al., [11], Hasancebi and Ulusay [7], Dikmen [9] and proposed correlations(without uncertainties). The statistical comparison is performed for all the 326 cases initially used for the correlation development. Fig.6 illustrates the scattering of predicted (calculated by different methods) versus observed shear wave velocity values.

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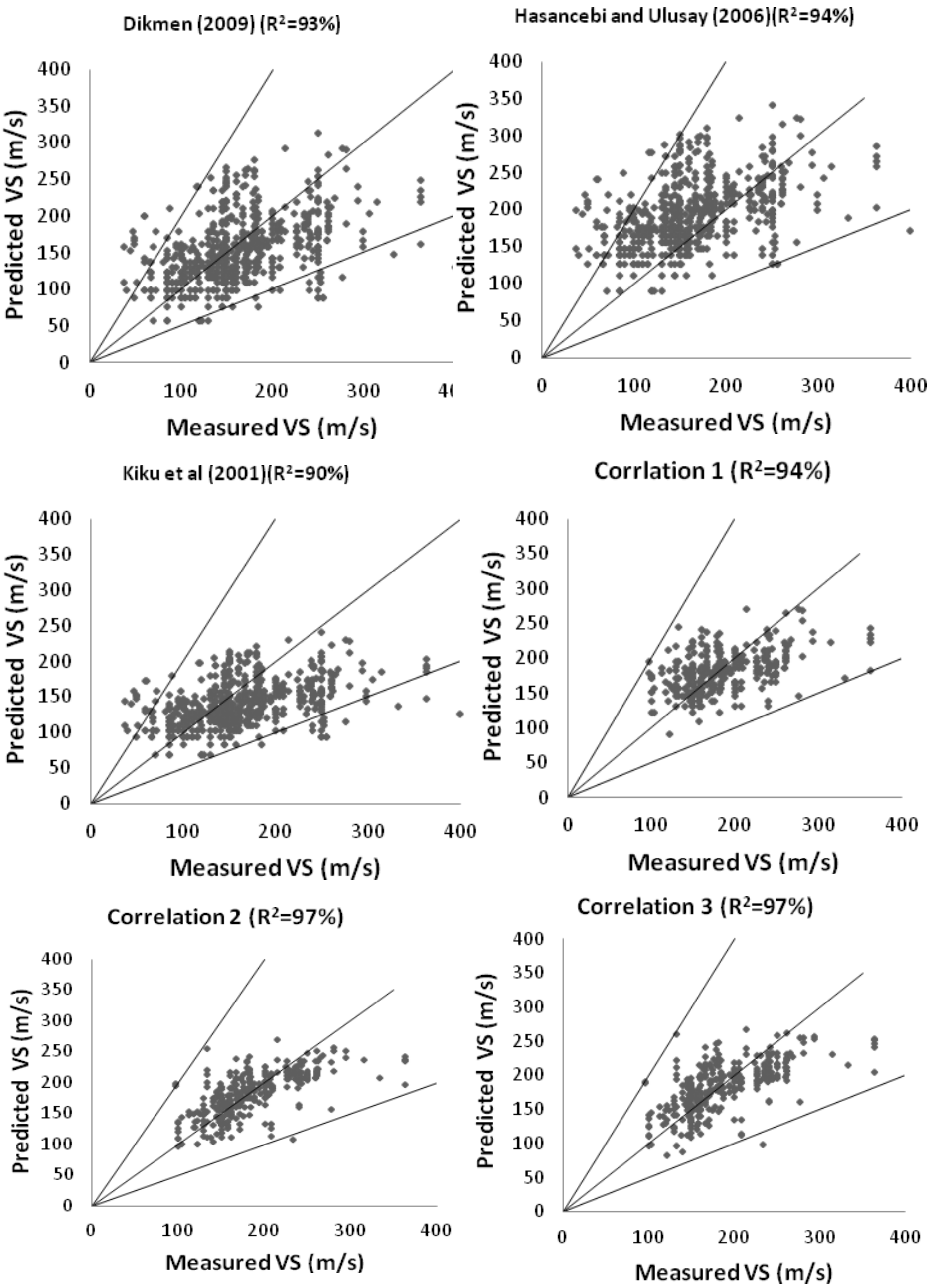

Fig.6. Estimated versus measured shear wave velocity by different methods

As shown in Fig.6, Kiku et al (2001), Hasancebi and Ulusay (2006)and Dikmen (2009) give higher Vs values than measured ones and these differences increase with up to Vs =100 and give lower Vs values over Vs =238. But according to proposed correlation, correlation 2 and 3 approximately were same in prediction and Vs can be higher accuracy and low scatter.

Conclusion

We attempted to deploy a system identification technique to develop uncertainty over shear wave velocity. The evolved robust optimization has been used to obtain a correlation for the prediction of Vs. A database of case histories consisting of 326 dataset from Turkey was compiled with the shear wave velocity. According to previous studies, correlations based on statistical methods introduced. In this paper, in order to consider uncertainty robust optimization method was used. The uncertainty was calculated as a new parameter.

The advantage of this approach include correlations developed without uncertainty (uncertainty=0%) equal to regression analyses. As shown in Fig.4. constant coefficient whose variations is high, is more sensitive. By estimation of uncertainty, the proper constant coefficient must be used to determine Vs. According to uncertainty of these data bases correlation 3 among proposed correlations for uncertain condition is preferred due to R^2 reduce rate by uncertainty as shown in Fig.5.

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Correlation 3 considers FC in order to limit SPT disadvantage usage in cohesive soils. The correlation 3 usage as shown in Fig.5 is proposed for all wide of fine content especially FC>75%.

Results obtained from this study and previous researchers reveal that empirical correlations derived from a local dataset should not be implemented for different sites with significantly varying features. Therefore, these proposed relationships should be used with caution in geotechnical engineering and should be out checked against measured VS.

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