Shear Strength-Dilation Characteristics of Silty and Clayey Sands

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Abstract

Sandy soils usually contain different amounts of fines like silt and clay, causing some changes to their shear strength and dilation characteristics. Bolton [1] conducted some experiments on ~~the~~ different sands and suggested a relation between the parameters of the soil shear strength. In this paper, some experiments were performed on fine contained sand and the extended Bolton's relation ~~was~~ has been proposed. In this paper, shear strength and dilation behavior of a pure sand mixed with different amounts of silt or clay fines were studied using direct shear test device (100*100*30 mm), and a total of 96 tests were carried out. The samples were prepared separately using clay and silt contents of 0, 10, 20 and 30% in different relative densities of 70, 80, 90 and 100%. They were tested under three surcharge pressures of 90, 120 and 150 kPa, under particle crushing threshold. Variations in shear strength, maximum friction angle, critical state friction angle and cohesion, as well as dilation angle were investigated by increasing in the mentioned

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amounts. The results demonstrate that shear strength, dilation angle, maximum friction angle decreased by clay content increase, however, they increase with increase in silt content. In addition, a new form of the Bolton's relation for fine contained sandy soils was presented.

Keywords: Direct shear test, shear strength, dilation, clay content, silty sand.

Introduction

The application of shear forces to sands cause changes in its volume, up to the critical state condition. Loose sands contract while dense sand expands during shearing. Critical state occurs when a sample experiences continuous shearing at constant shear stress without changes in volume (Poulos et al., [2]; Haeri and Hamidi, [3]), and at this point, the dilation angle reaches zero. In nature, soils are usually deposited in mixed form that is, sand-silt or sand-clay particles, therefore it is interesting to investigate and compare the shear strength and dilation characteristics of sand with that of sand-silt or sand-clay mixtures.

A number of studies have been previously carried out on shear strength-dilation relation of granular soils. Bolton [1], [4] evaluated and analyzed the data obtained from triaxial and plane strain tests on 17 different sands at different relative densities and confinements or surcharge pressures. All confinements were lower than 150 kPa, which is the threshold of particle crushing for silica sands. According to the results, Bolton [1] presented the following empirical equation that expresses the effect of dilation angle on shear strength of sandy soils:

$$\phi_{max} = \phi_{cv} + 0.8\Psi_{max} \tag{1}$$

Where, ϕ_{max} and ϕ_{cv} are maximum and critical state friction angles, respectively. This relationship can be rewritten for plane-strain tests as:

$$\phi_{max} - \phi_{cv} = 0.8\Psi_{max} = 5I_R \tag{2}$$

Moreover, for triaxial condition as follows:

$$\phi_{max} - \phi_{cv} = 0.8\Psi_{max} = 3I_R \tag{3}$$

Where, I_R is dilatancy index and can be defined as:

$$I_R = -\frac{10}{3}(\frac{d\varepsilon_v}{d\varepsilon_1})_{max} \tag{4}$$

In this equation, $d\varepsilon_v$ is the increment of volumetric strain and $d\varepsilon_1$ is the increment of major principal strain.

Later, some researchers evaluated the addition of gravel to sand and proposed new forms of Bolton [1] equation for sand-gravel mixtures (Simony and Houlsby, [5]; Hamidi et al., [6-8]. Hamidi and Soleimani [8] performed extensive studies on cemented gravely sands and extended the Bolton [1] equation for cemented gravely sand as follows:

$$\phi_{max} - \phi_{cv} = 1.2(e_{min,uncemented} - e_{min,cemented})\Psi_{max} \tag{6}$$

A number of experimental studies have been performed to investigate the mechanical behavior of fine contained sands. Hight et al. [9] investigated the behavior of clayey sands under static and cyclic loading conditions and found that an increase in fine fraction up to 40% decreases the dilatancy and after that, the soil behavior is controlled by finer matrix. Pitman et al. [10] conducted some undrained triaxial tests and found that brittleness decreases with increase in plastic or non-plastic fine content. Salgado et al. [11]

investigated the effect of silt addition to Ottawa sand. They included silt contents of 5 to 20% to the sandy soil and determined the strength parameters in different relative densities using triaxial tests. They continued the tests up to 30% of axial strain to consider the critical state of soil in large axial strains. Based on the results, it was concluded that an increase in silt content and relative density increased the maximum and critical state friction angles of sandy soil.

Naeini and Ziaie Moayed [12] discussed the effect of silt content on the undrained strength of loose sands using the cone penetration test results. Olmez [13] performed series of physical tests on the clay-sand mixtures and studied the changes in the basic characteristic of the soil, such as void ratio, grain size distribution and Atterberg limits with increase in clay content. It was concluded that the characteristics of the volume change of the samples considerably varies with kaolin content up to 20%. Gupta and Trivedi [14] examined the effect of non-plastic fines on the behavior of loose sand by triaxial shear and plate load tests and analyzed the effect of increase in fines on the compressibility and internal friction angle of the soils. The results of the laboratory tests showed that the maximum and minimum void ratios of clean sand decreased as fine content increased from 0 to 20% but it would increase again if fine content exceeded 20%. The results also indicated that the angle of internal friction and shear strength decreased with the addition of fines due to compressibility of the fine particles.

Chakraborty and Salgado [15] performed triaxial shear and plan-strain tests on the Toyoura sand at initial confining stresses lower than 197 kPa. They also examined the changes of dilation angle and maximum internal angle, and their variations with relative density and confining stress. They analyzed the results of triaxial and plain strain tests on Toyoura sand for the initial confining stresses ranging from very low to 197 kPa to examine the dependence of dilatancy and friction angle on relative density and confining stress. The minimum confining stress was as low as 2 kPa in case of triaxial compression and 4.9 kPa in case of plain strain tests. They proposed correlations of critical state and peak friction angles besides the state parameters of soil based on the Toyoura sand data using test results. Pakbaz and Moqaddam [16] analyzed the effect of clay addition on the behavior and shear strength of sandy soils and concluded that the addition of 15-40% clay fines to the sand considerably causes changes in its shear and dilation behavior. Increase in the percentage of fines (15% to 40%) decreases its internal friction angle and shear strength. Ojha and Trivedi [17] analyzed the shear strength parameters of silty sand. Some triaxial experiments were conducted on samples containing up to 25% silt content, and the changes of shear strength due to the increase in silt percentage was examined. Then, the effect of silt content on the internal friction angle, minimum and maximum void ratios, and efficient size of the fines was also studied. Yu [18] studied the stress-dilatancy behavior of sand incorporating particle crushing. They found that particle breakage increases with the increase of axial strain and confining

pressure. Particle breakage was also observed during consolidation and denser samples showed more particle breakage compared to the looser ones.

Patil et al. [19] investigated the behavior of unsaturated silty sands using triaxial tests and showed the increase in internal friction angle with increase in silt content. Recently, Amri et al. [20] investigated the behavior of clayey sand using consolidation tests besides unconfined compression, triaxial shear, XRD and Atterberg limit tests and reported the threshold sand content of 30% as a limit to reduce the swelling potential of mixture.

Reviewing the technical literature shows in most of the studies, the focus was more on the sand mixture with clay and silt and the changes of the shear strength parameters, as well as the dilation of these fines and few studies were concerned with the relationship between the strength parameters and dilation of these mixtures. In most sandy deposits, like the coasts of Babolsar in north of Iran studied herein, the silt or clay particles exist as a dominant part of soil. Therefore, it seems necessary to investigate the strength behavior of this category of soils in detail. In addition, in order to consider the strength and dilation behavior of these mixtures, an extension of Bolton's (1986) relation is presented in the present study. For this purpose, samples with relative densities of 70, 80, 90 and 100% with clay or silt contents of zero, 10, 20 and 30% were prepared and shear behavior under overburden pressures of 90, 120 and 150 kPa (less than particle crushing) was investigated. The direct shear apparatus is a useful tool for investigation of the shear strength

and dilation characteristics of the soils because the variation of both shear strength and vertical displacement can easily be performed with horizontal or shear displacement. However, for the triaxial tests, the volumetric strain is an indicator of the dilation induced in the sample. As a result, direct shear tests were selected to investigate the behavior of samples in present study. The main objective of this study is to consider the relationship between strength and dilation for silty sand and clayey sand.

Material

Clean quartz beach Babolsar sand from the shores of Caspian Sea in North Iran was used as the base soil. Grain size distribution is shown in Figure 1 and physical properties are shown in Table 1. The soil was classified as poorly graded sand, SP following the unified system of soil classification. The kaoline clay and silty soil, which was obtained from Karaj River near the city of Tehran, were used as the fines. Samples with clay or silt contents of 0, 10, 20, and 30% were prepared. Liquid and plastic limits, as well as the plasticity index of the fines are shown in Table 2.

Table 1. physical properties of Babolsar sand

Name	SP
D_{10}(mm)	0.17
D_{50}(mm)	0.25
G_s	2.74
C_u	0.63
C_c	1.15

Table 2. Atterberg Limits of kaolin and Karaj river silt

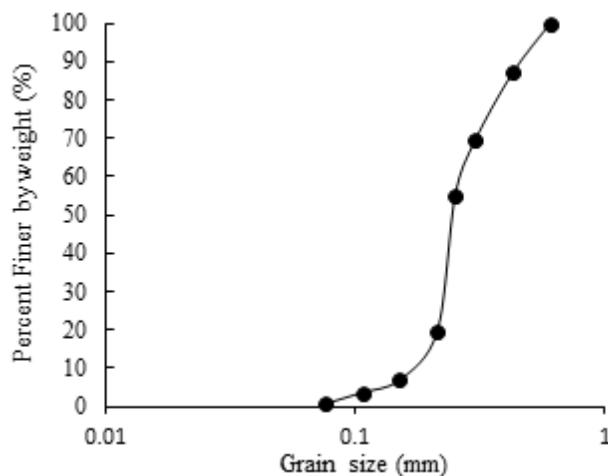| Fines	Liquid limit (LL%)	plastic limit (PL%)	Plasticity index (PI%)
Kaolin clay	39	25	14
Karaj river silt	25	23	2

Figure 1. Grain size distribution of Babolsar sand

Figure 2 shows variations of the minimum and maximum unit weights of silty sand with silt content. The maximum and minimum unit weights were determined according to ASTM D-4253 and ASTM D-4254, respectively. An important point to note is that ASTM limited the maximum fine content to 15% in these tests, especially for the maximum unit weight. Due to this limitation, different scholars used different methods to deal with these challenges; many proposed using the same standard, regardless of its problems. To increase the reliability, Yang et al. [21] utilized the standard proctor test. However, testing was coupled with error at 15 to 30% fine contents. As shown in Figure 2, maximum and minimum

unit weight occurred for silt by 40%, and then the maximum and minimum unit weights experienced a smooth descending trend. The reliability and accuracy of this chart in silt contents above 60% is low, but due to the focus of the present research on fine contents up to 30% and lower, the values can be assumed as acceptable.
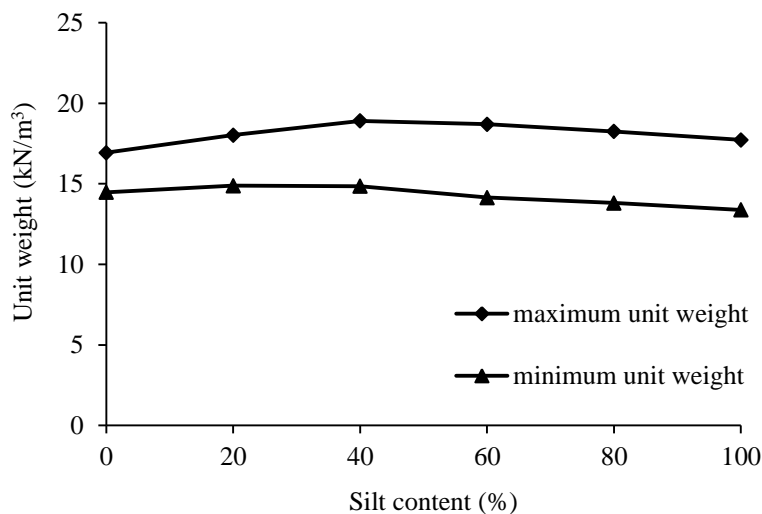

Figure 2. Variation of minimum and maximum unit weights with silt content

For silty fines, shaking table test was carried out in present study to evaluate the maximum unit weight of soil. However, to calculate the maximum unit weight, especially for the mixture of clay-sand, it is not possible to use the same procedure as silty sand, and Equation 7 can only be used for 0 to 10% clay content. For samples containing 20 to 30% clay content, Equation 8 was used instead, based on the maximum unit weight obtained from the modified proctor density test (ASTM D-1557). Tables 3 and 4 show the minimum and

maximum unit weights besides specific density for the samples containing silt and clay fines, respectively.

$$D_r = \frac{\frac{1}{\gamma_{min}} - \frac{1}{\gamma}}{\frac{1}{\gamma_{min}} - \frac{1}{\gamma_{max}}} \tag{7}$$

$$R_d = \frac{\gamma}{\gamma_{max}} \tag{8}$$

A 100mm×100mm×30mm direct shear test was used in present study. Samples were prepared at different relative densities of 70, 80, 90 and 100% and required weights of the soil were determined based on Equations (7, 8) based on the fine type and fine content. Each sample was poured in three layers in a shear box and each layer was made to reach the desired density by a steel hammer. All the samples were initially prepared at 8% water content for shaking to obtain better compaction. This amount was sufficient for all clay contents considered in present study. To avoid particle crushing, vibration table was used instead at higher relative densities. After some hammer blows, the shear box was placed on the table and vibrated under a surcharge load to attain the desired unit weight. Rate of shear loading for pure sandy samples was 1 mm/min, while it was reduced to 0.2 and 0.1 mm/min for samples containing silt and clay fines, respectively. This value was considered to be consistent with other researches in this filed like the study of Olmez [13] that used the rate of 0.12 mm/min or Pakbaz and Moqaddam [16] that used the rate of 0.1 mm/min in their experiments. In fact, for silty sand or sand-clay mixtures, the rate of loading was decreased to prevent generation of pore pressure within the sample. All samples were saturated and for full saturation, water content was measured in different time intervals

of 12, 24, 36, 48 hours. The samples were considered saturated when the measured moisture content between two consecutive intervals became similar. For high clay contents, a minimum submerging time of 96 hours was considered to achieve full saturation as emphasized in ASTM-D3080. The surcharge pressures were selected as 90, 120 and 150 kPa in consistence with the values of Bolton [1] and lower than particle crushing limit for silica sands. Other variables considered in the experiments are listed in Table 5.

A number of 96 direct shear tests were performed in the present research. During each test, the shear force, besides horizontal and vertical displacements, were measured continuously up to the horizontal displacement of 12 mm using calibrated gauges.

Table 3. Minimum and maximum unit weights and specific density for clean sand and silty sand

Silt content (%)	$\gamma_{d\ max}\ (\frac{kN}{m^3})$	$\gamma_{d\ min}\ (\frac{kN}{m^3})$	G_s
0	17.2	14.6	2.740
10	17.8	14.7	2.734
20	18.0	14.9	2.728
30	18.8	14.9	2.722

Table 4. Minimum and maximum unit weights and specific density for clean sand and clayey sand

Clay content (%)	$\gamma_{d\ max}\ (\frac{kN}{m^3})$	$\gamma_{d\ min}\ (\frac{kN}{m^3})$	G_s
0	17.2	14.6	2.740
10	18.8	14.4	2.731
20	20.2	-	2.722
30	21.2	-	2.712

Table 5. Variables of testing program

Variables	Unit	Value
Relative density	%	70, 80, 90, 100
Silt content	%	0, 10, 20, 30
Clay content	%	0, 10, 20, 30
Normal stress	kPa	90, 120, 150

Analysis of test results

Shear stress-shear displacement curves were plotted as indicated in Figures 3 and 4. According to the figure, shear strength of sandy soil increased with increase in silt content and decreased with increase in clay content. Table 6 also shows a summary of the calculated data determined for all the experiments. These data have been implemented in derivation of empirical relations.

1. Peak friction angle

Figure 5 shows the variation of peak friction angles in different silt and clay contents versus relative density. As can be seen, the effect of clay and silt fines on sandy soil friction angle is quite reverse. This implies that by increasing the percentage of silty fine in sandy mixtures (up to 30%), the maximum friction angle increases, however, increase in clay fine percentage in sandy mixture (up to 30%) decreases the maximum friction angle.

According to Figure 5, an increase in the relative density increases the internal friction angle for all silt or clay contents. This is due to the reduction of void ratio between the grains and associated increase in friction. Investigating 4 curves of friction angle changes in different silt contents and relative densities reveal that in a similar relative density, the internal friction angle increases with silt content

Table 6. Values of parameters implemented for derivation of empirical relationships for all the tests

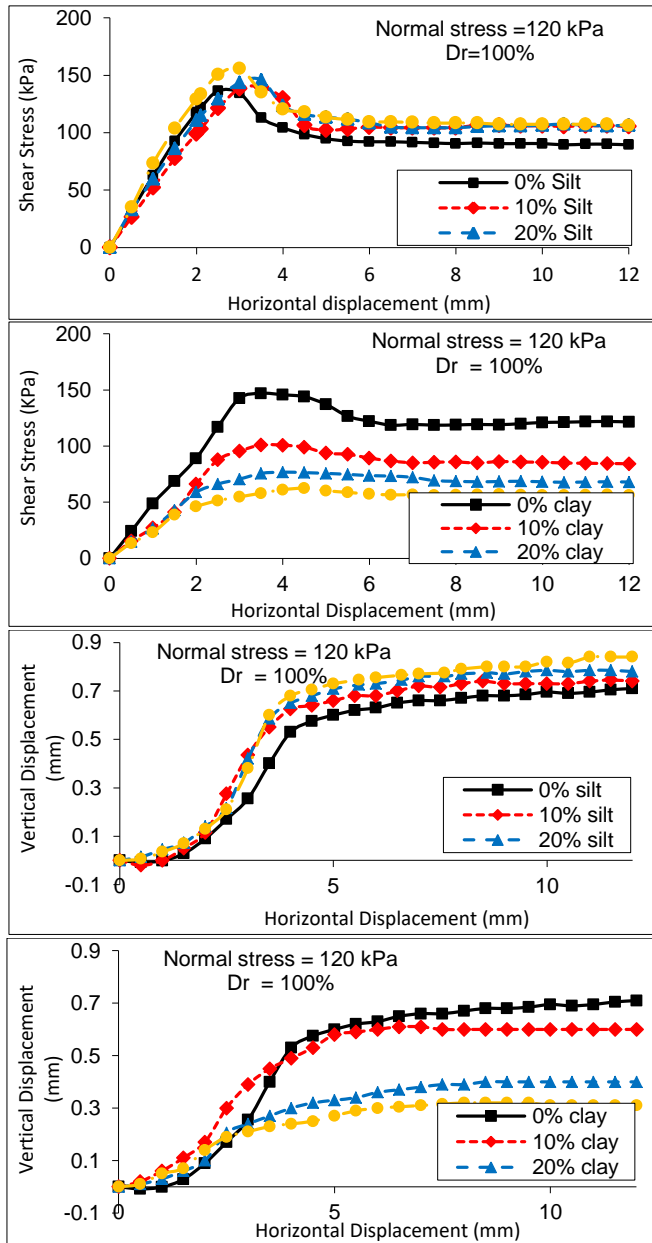| Fine content	Fine type	Relative density (%)	Maximum friction angle	Critical state friction angle	Maximum dilation angle
0	-	70	20	14.75	6.3
		80	28	23.3	8.3
		90	35	27.15	10.8
		100	44	35.25	13.2
10	silt	70	25.3	14.77	9.6
		80	32.7	21.42	10.6
		90	39.1	26.94	12.5
		100	48.4	33.5	15.5
20	silt	70	24	16.1	11
		80	31.2	23.3	13
		90	37.3	27.3	15
		100	46	35.2	17
30	silt	70	22	19.56	12
		80	30	25.6	14.5
		90	37	30.7	18
		100	44.6	37.5	21.7
10	clay	70	18	14.77	5.1
		80	24	19.8	7.9
		90	30	25.1	9.6
		100	37	31.5	10.6
20	clay	70	15	11.7	4.4
		80	20	16.1	7.5
		90	24	19.8	8.5
		100	28	23.3	9.1
30	clay	70	13	9.9	3.9
		80	16	12.5	4.6
		90	20	16.1	6
		100	22	17.8	6.8

Figure 3. Direct shear test results in the sand mixed with different percentage of clay and silt in relative density of 100% under the surcharge pressure of 120 kPa
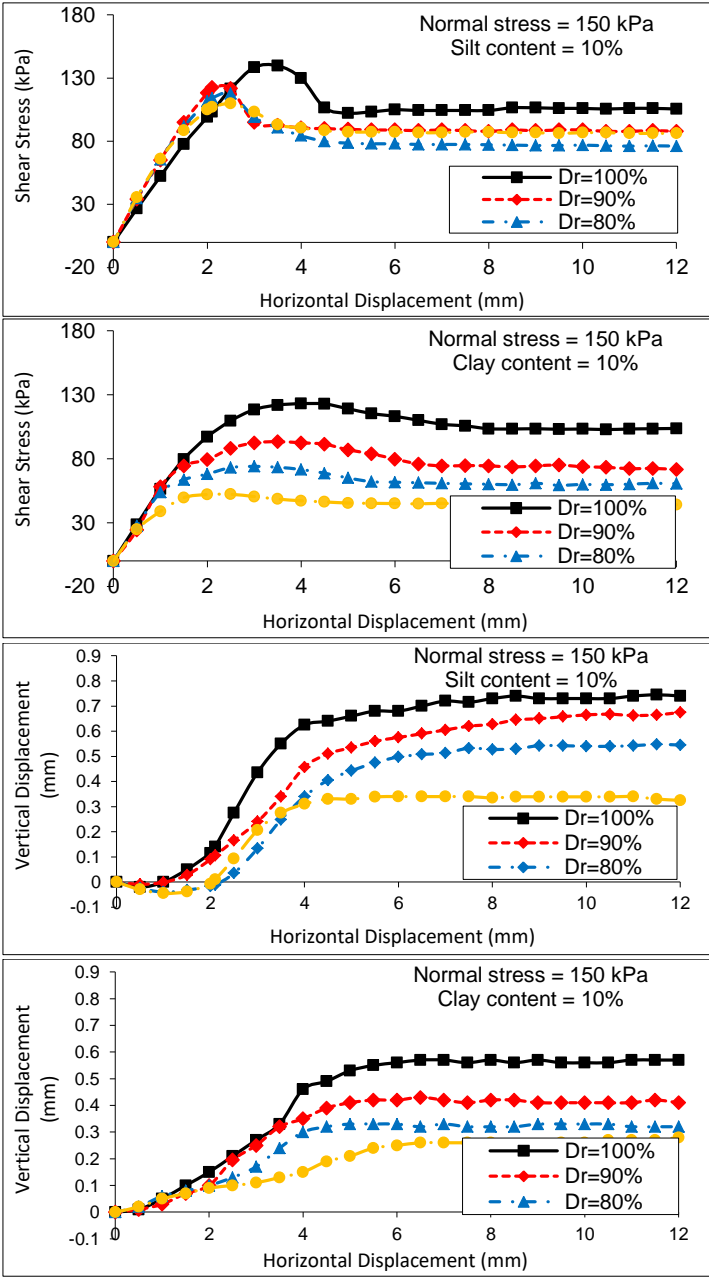
Figure 4. Direct shear test results in the sand mixed with 10% clay and silt and different relative densities under the surcharge pressure of 150 kPa

for 2 or 3 degrees. Moreover, in similar silt content, an increase in the relative density increases the internal friction angle for 7 to 9 degrees. For clayey sands, an increase in clay content for a constant relative density decreases the internal friction angle from 2 to 5 degrees. In addition, in similar clay contents, an increase in relative density increases the friction angle from 3 to 7 degrees. Based on Figure 5, the reduction in friction angle is not constant for different clay contents. For example, a 10% increase in relative density of mixture with 10% clay increases the friction angle about 7 degrees. However, for mixtures with 30% of clay, this value is about 3 degrees. The observed trend of behavior is in accordance with the results of Salgado et al. [11] on silty sand and Olmez [13] for clayey sand.
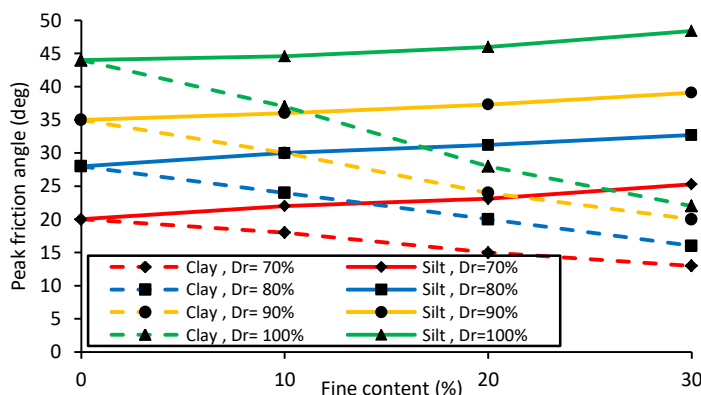

Figure 5. Variation of peak friction angle with different fine contents

2. Critical state friction angle

Critical state friction angle represents the minimum shear strength of the soil as it shears in constant volume, effective normal stress, shear stress and constant rate of axial strain (Poulos [22]). Critical state friction angle can be considered as one of the intrinsic

characteristics of materials whose value depends on the mineralogy, type, and shape of grains. Figure 6 displays the variation of critical state friction angle for sandy soils with fine content in different relative densities. Comparing these curves shows that increase in silt content increases the critical state friction angle. However, the increase of clay fine percentage in sandy soil decreases the critical state friction angle. In addition, increase in relative density for both fine types increases the critical state friction angle, however, the rate of decrease is more pronounced for clay fines than the rate of increase for silty one.

Comparison of Figures 5 and 6 shows that the trend of variations for the critical state friction angle with clay content, silt content and relative density are almost similar to the maximum friction angle of soil.
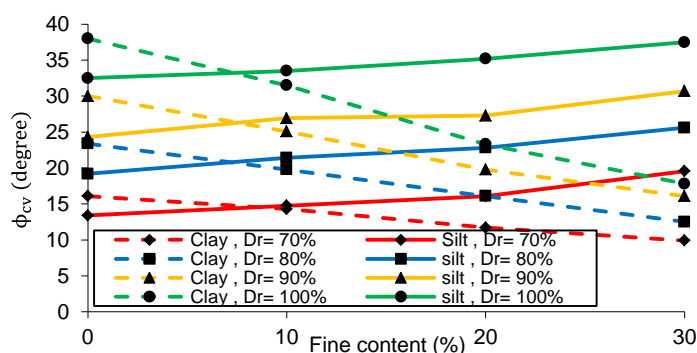

Figure 6. Variation of critical state friction angle with different fine contents

3. Cohesion intercept

Figure 7 shows the variation of cohesion intercept with clay content in different relative densities. As expected, cohesion intercept increased with clay content and relative density. However, it is

obvious that relative density is more effective on the rate of increase in cohesion intercept than clay content.

According to Figure 7, cohesion intercepts increase with an increase in relative density and clay content. The maximum rate of increase with relative density exists in clay content between 10 to 20%. For a constant relative density, an increase in clay content from 0 to 30% increases the cohesion intercept about 7kPa. In addition, an increase in relative density from 70 to 100% increases the cohesion intercept about 10kPa. It should be noted that for silty sand, the change in cohesion intercept was not meaningful due to the insignificant cohesion of silt as a fine-grained soil.

It should be noted that cohesion intercept was assumed as zero for silty sand mixtures in the present study. Comparison of the variation of cohesion intercept with clay content obtained from present study and Olmez [13] shows good consistency between the results.
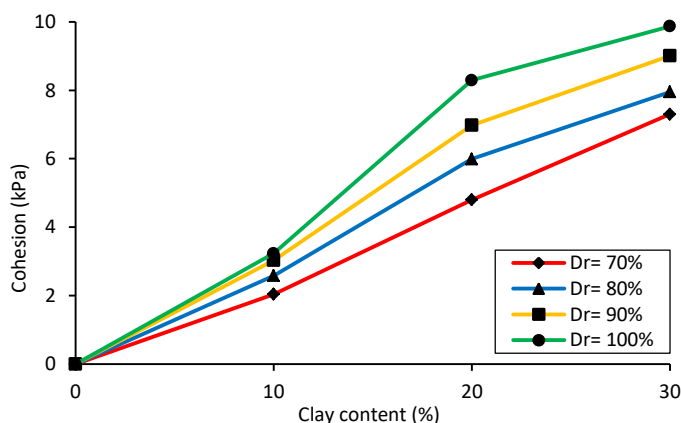

Figure 7. Variation of cohesion intercept with clay content in different relative densities

4. Dilatancy

Angle of dilation (ψ) can be defined in direct shear test according to the following equation:

$$\psi = \tan^{-1}(\frac{\Delta v}{\Delta u}) \tag{9}$$

Where Δv is the change in vertical displacement and Δu represents the horizontal or shear deformation of a sample in each loading step. The angle of dilation was calculated during the tests and the maximum established value was considered as maximum dilation angle.

Figure 8 shows the typical dilation-shear displacement variation during direct shear test. According to the figure, sudden and irrational skips can be observed in limited number of points during the tests that increased the dilation angle of the samples. The presence of sudden skips in specific points can be explained with respect to the fact that in direct shear test, series of general distortion is induced in shear plane, which result in sudden skips in dilation as shown in Figure 9.

In order to eliminate the skip points of dilation, smooth and continuous curves by MATLAB was used instead as shown in Figure 10. After smoothing of the dilation curves, dilation angle was calculated using Equation (9). The same procedure has been implied in a number of other studies on shear strength-dilation characteristics of sand-gravel mixtures [23-25] and cemented sands [26-27].

Figure 1 shows the variation of dilation angle calculated from smooth dilation-shear displacement curves with silt or clay content. Comparing the curves shows that an increase of clay fine percentage in sandy mixtures decreases the dilation angle but increase in silty fine content increases the dilation angle of the soil. This different behavior results from the cohesion induced by clay fines that turns the clay and sand mixtures into a unite pulp and decreases its dilation. However, there is no cohesion in silty mixtures, which allows the sand and silt materials to move more freely, resulting in more dilation compared to clayey sand. The observed pattern of behavior in the present study is consistent with the results of Salgado et al. [11].

For both clayey and silty fines, the increase in relative density increases dilation angle of the soil. However, for the similar increase in relative density, the net increase in dilation angle of silty soil is more compared to the clayey soil.

5. Shear strength-dilation relations

Bolton [1] provided an empirical relation between shear strength and dilation of sandy soils based on experiments conducted on 17 different clean sands. To calculate this relation for sandy soils containing clay fines, graphs of $\phi_{max} - \phi_{cv}$ against Ψ_{max} are drawn based on the results of the present study as shown in Figure 12. As can be seen, contrary to Bolton's relation, there is an intercept in these relations for clayey fine due to the induced cohesion.
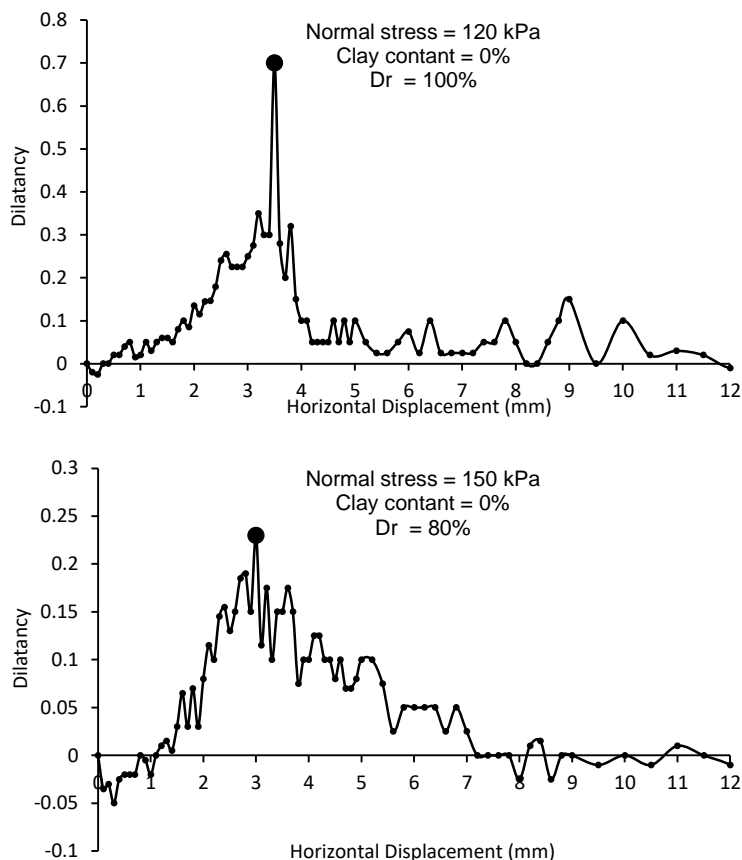
Figure 8. Typical variation of dilatancy with horizontal displacement

In addition, it can be seen that there exist a unique relation in this space for different clay contents and relative densities; however, the intercept gradually decreases as the overburden stress increases from 90 to 150 kPa. For the range of normal stresses (90~150 kPa), relative density of (70~100%) and clay content of (0~30%) in present study, the following equation can be proposed for clayey sand:

$$\phi_{max} - \phi_{cv} = 0.3\,\psi_{max} + 2° \qquad (10)$$

As it can be observed, coefficient of 0.8 in Bolton's equation for pure sand is reduced to 0.3 in this relation for clayey sand; however, an intercept is added to the relation, which was not previously proposed in Bolton's relation.
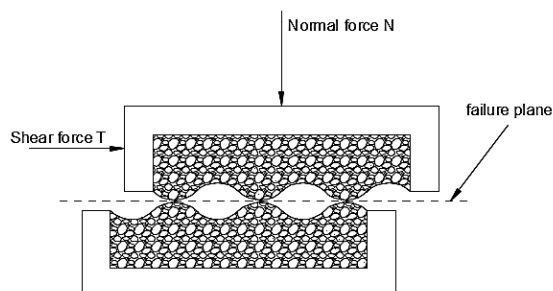

Figure 9. Increase in dilation due to the non-uniformity of sample surfa

Figure 13 shows Bolton's relations for silty sand in different surcharge pressures. Contrary to clay fines, Figure 13 displays that the relation is quite dependent on silt content. Bolton's coefficient decreases from 0.8~0.9 for zero silt content to about 0.4~0.6 for 30% silt content. Also, Figure 13 demonstrates zero intercept with good regression coefficient for all silt contents similar to Bolton's [1] relation. The Figure also shows the comparison between the measured results and predicted values based on Equation (10) for clay fines. According to the Figure 14, the data are all placed around the 1:1 line with a difference less than 0.7 degrees, which shows the consistency of the suggested relation with the experimental results.
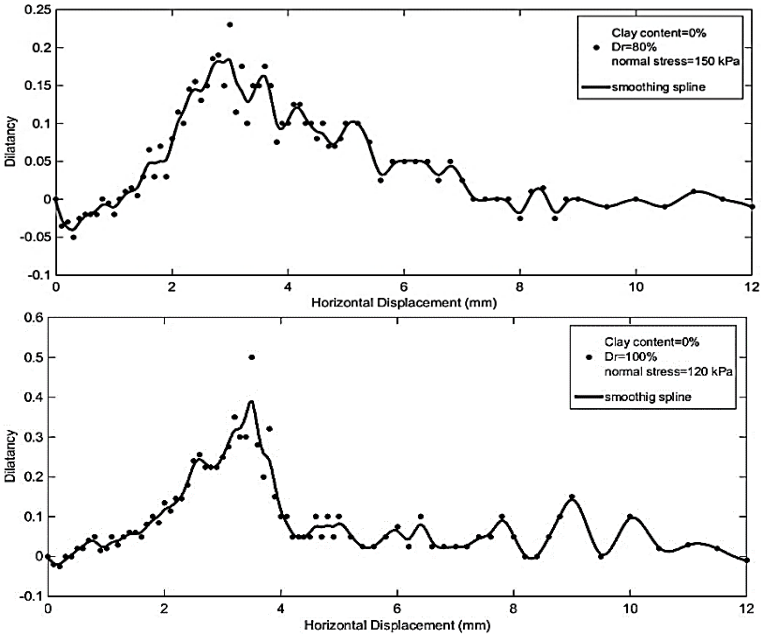
Figure 10. Smooth dilatancy-horizontal displacement curves for different relative densities
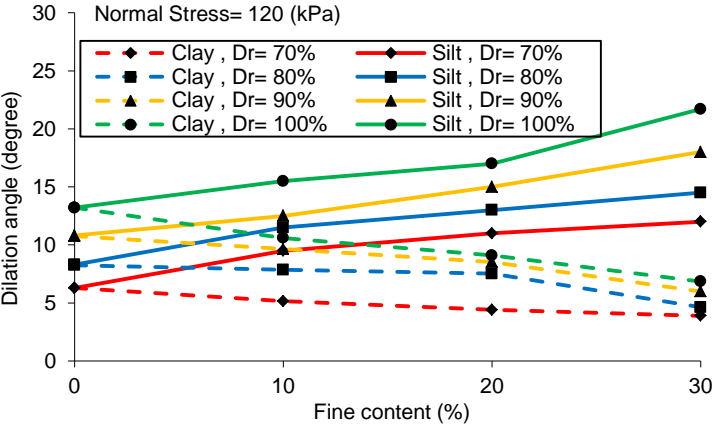

Figure 11. Variation of dilation angle with fine content in different relative densities

For the 90 kPa chart:

Normal Stress= 90 kPa
clay

$$\phi_{max} - \phi_{cv} = 0.27\,\psi_{max} + 1.95$$

$$R^2 = 0.95$$

For the 120 kPa chart:

Normal Stress= 120 kPa
clay

$$\phi_{max} - \phi_{cv} = 0.29\,\psi_{max} + 2.03$$

$$R^2 = 0.94$$

For the 150 kPa chart:

Normal Stress= 150 kPa
clay

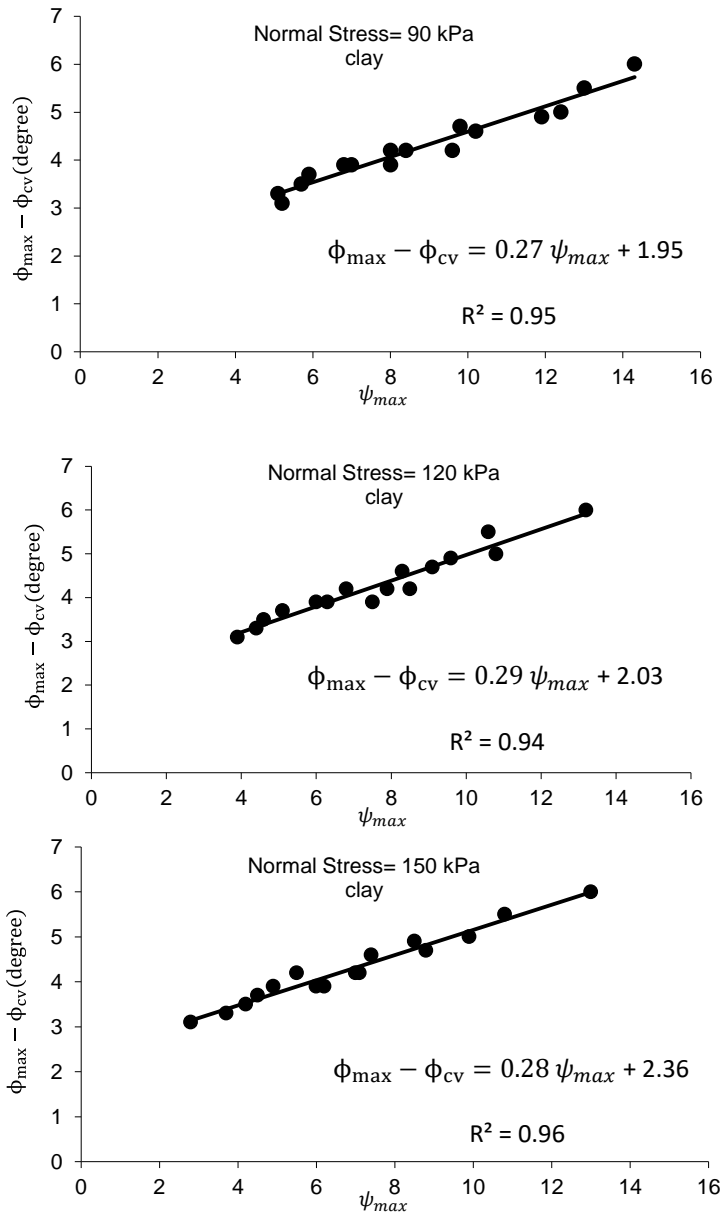$$\phi_{max} - \phi_{cv} = 0.28\,\psi_{max} + 2.36$$

$$R^2 = 0.96$$

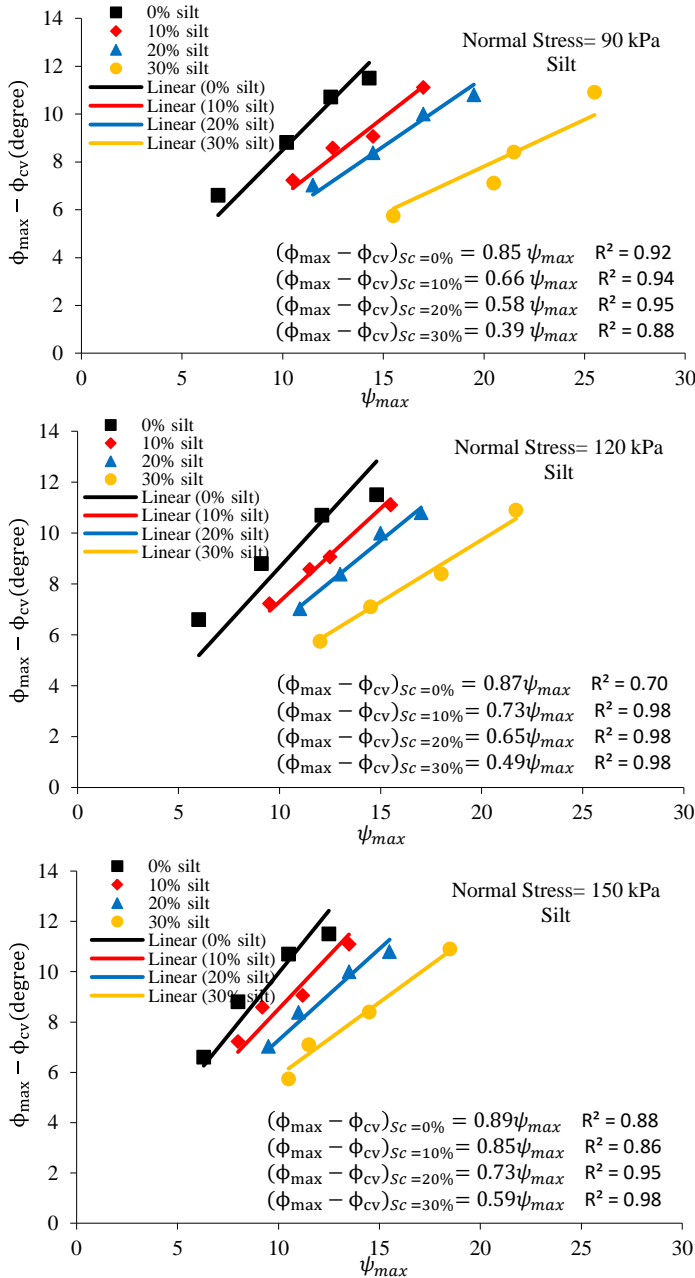**Figure 12. Relation between $\phi_{max} - \phi_{cv}$ and maximum dilation angle for clay**

Figure 13. Relation between $\phi_{max} - \phi_{cv}$ and maximum dilation angle
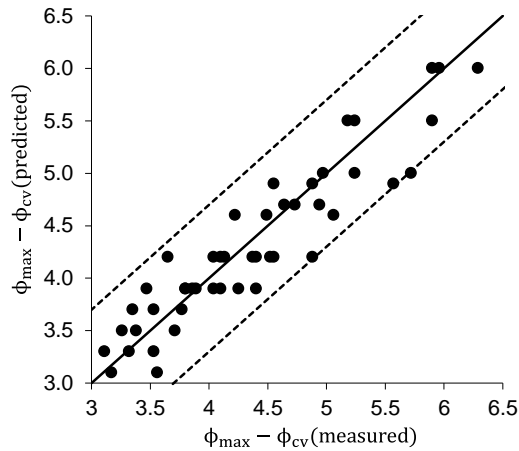for silty fine in different surcharge pressures

Figure 14. Comparison of measured and predicted values of $\phi_{max} - \phi_{cv}$ based on empirical equations for clayey sand

Summary and conclusions

In this study, 96 direct shear tests were conducted to determine the effect of increasing clay and silt percentage besides the relative density or relative compaction and overburden pressure on Bolton's [1] relationship. The samples were separately made in 0, 10, 20, 30% clay or silt content, hammered in 70, 80, 90, 100% relative density or relative compactions and tested under three overburden pressures of 90, 120, 150 kPa. According to the tests, the following results were obtained:

- Increase in clay content in sandy soil decreased the shear strength and increase in silt percentage caused its increase in the range of fine contents considered in the present study.

- Increase in clay content decreased the dilation angle but increase in silty fine increased the dilation angle in the samples. The

difference in behavior was related to the cohesion induced due to the increase in the clay content.

- A new form of Bolton's relation was proposed for clayey sand. According to the new relation, there existed an intercept in equation, which depended on the amount of surcharge pressure. In addition, the coefficient dropped from 0.8 to 0.3 in clayey sands. For silty sands, the intercept was zero but the coefficient was strongly dependent on silt content. Its value decreased with increasing silt percentage.

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