

# The New Empirical Formula to Estimate the Uniaxial Compressive Strength of Limestone; North of Saveh a Case Study

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## Abstract

Uniaxial compressive strength is one of the most important features to describe the resistive behavior of rocks which is used as an important parameter in the design of structures especially underground openings. The determination of this parameter using direct methods such as ordinary uniaxial compressive strength tests is costly and time consuming, and also sometimes preparation of standard samples in many rocks is difficult. In such cases, the implementation of the simple non-destructive tests and using empirical relations can increase the evaluation speed and reduce costs. These relations even regional or local (for example within a geological formation or a single lithology) can help in the estimation of these parameters in order to be

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used in geotechnical projects. In this study, samples of existing limestones in north of Saveh were prepared and uniaxial compressive strength, point load, Schmidt hammer and shear wave velocity tests have been performed. Then, by the statistical evaluations of the results, the empirical relations between uniaxial compressive strength and the results of other tests have been obtained. Finally, to evaluate derived relations, they have been compared with the same relations. The comparison between the predicted and observed values of uniaxial compressive strength represents the validity of obtained empirical relations. The application of the proposed relations for limestones in the study area and those with similar geological conditions will provide acceptable results.

**Keywords:** Uniaxial compressive strength, Point load, Schmidt hammer, Shear wave velocity, limestone, Correlation

## Introduction

These days, determination of the required geotechnical parameters in the design of structures located on bedrock (including rock foundations, tunnels, intact rock and rock mass classification and etc.) is one of the major issues in civil and mining engineering. One of these parameters that are widely used in the design of underground structures is uniaxial compressive strength (i.e. UCS) of rock. This parameter is determined directly based on the ASTM-D2938 standard through tests on intact rock samples. However, implementation of this

test despite simplicity is costly and time consuming [1]. One of the limitations of this test is the difficulty of the standard sample preparation.

Since the calcareous rocks are one of the most abundant sedimentary rocks in the earth's crust, they should be evaluated based on their applications in the engineering projects (e.g. use as a borrow material, rock foundations, rocks structures, underground structures such as tunnels and subway and etc). Therefore, development of the empirical relations to assess the geotechnical characteristics of the rocks by physical properties and simple experiments is necessary. Many researchers have used indirect tests such as Schmidt hammer and point load tests to define uniaxial compressive strength of rocks. Considering the abundance of limestone in Iran specially around the metropolitan regions, using simple and cheap experiments including Schmidt hammer hardness, point load index and speed shear wave tests, uniaxial compressive strength of limestone rocks in north of Saveh is evaluated and the relations between the results of these experiments and uniaxial compressive strength are presented as the empirical relations.

In order to estimate the uniaxial compressive strength of rocks using other parameters and indices tests many investigations have been performed. Accordingly, different experimental relations depending on the type of rocks, the circumstances and method of testing are proposed. Deer and Miller (1966) using Schmidt hardness

test on 22 different lithology achieved empirical relation for indirect determination of uniaxial compressive strength [2]. Katz et al. (2000) suggested a relation to determine compressive strength using Schmidt hardness, physical and chemical properties of limestone and sandstone. After that, many researchers developed several empirical relations to assess indirect uniaxial compressive strength on the rocks for different areas [3]. One of the most accurate relations is Dincer et al. (2004) empirical relation [4]. Torabi et al. (2010) recommended a relation through the implementation of point load, Schmidt hammer and uniaxial compressive strength experiments on the sandstone, siltstone and shale rocks to determine the uniaxial compressive strength with the amount of Schmidt hardness and point load index in some relations [5].

Tondon and Gupta (2014) presented a relation through the implementation of point load, Schmidt hammer, shear wave velocity and uniaxial compressive strength tests on quartz, granite, gneiss and dolomite rocks to estimate compressive strength based on test results [6]. Bednarik et al. (2014) studied physical and mechanical properties of Leitha Limestone, broadly used as historical building materials in Eastern Austria, while Al-Omari et al. (2015) investigated on the petro-physical and mechanical properties of two porous limestones used in the construction and restoration works at the castle of Chambord in France, which is a UNESCO World Heritage site [34,

35]. Some empirical relations in order to estimate the uniaxial compressive strength are presented in Table 1.

Tugrul and Zarif (1999) related the uniaxial compressive strength of limestone to the shear wave velocity with a low correlation coefficient [7]. After that, several researchers developed this relation. Sharma and Singh (2009) relation for loose rocks and then Khandelwal and Singh (2009) relations for shale, sandstone and coal are the most accurate ones [8, 9]. Yagiz (2011) presented a relatively accurate relation between uniaxial compressive strength and shear wave using shear wave velocity and uniaxial compressive strength for all kinds of travertine rocks, loose limestone and dolomitic limestone [10]. Diamantis et al. (2011) represented a relatively close relation to estimate the uniaxial compressive strength using shear wave velocity for peridotite [11]. Minaian and Ahangari (2013) offered a relatively accurate relation to determine uniaxial compressive strength for conglomerate rocks by shear wave velocity and uniaxial compressive strength test results [12]. Pappalardo et al. (2016) presented a relatively accurate relation between uniaxial compressive strength and shear wave velocity, elastic modulus and permeability for limestone [36]. Other relations also suggested by other researchers (Table 1).

Deer and Miller (1966) developed a relation to determine the uniaxial compressive strength using a point load index for sandstone [2]. Then, other researchers began using the point load index to determine the compressive strength. Sabatakakis et al. (2009) conducted an experiment with point load on samples of sedimentary

rocks such as sandstone and limestone to determine the uniaxial compressive strength for three ranges of the point load index including lower than 2 MPa, between 2 and 5MPa and greater than 5MPa [13]. Cobanoglu and Beren (2009) performed point load and uniaxial compressive strength tests on sandstone, limestone and cement, then offered a relation to estimate the uniaxial compressive strength using the point load index [14]. Singh et al. (2012) presented a relation with a relatively high correlation coefficient using uniaxial compressive strength test results for each of the schist, sandstone, limestone and dolomite [15]. Li and Wong (2013) developed two relations with high accuracy for sandstone and silty rocks [16] (see Table 1).

**Table 1. Some relationships to determine the UCS by Schmidt hardness, shear wave velocity and point load index**

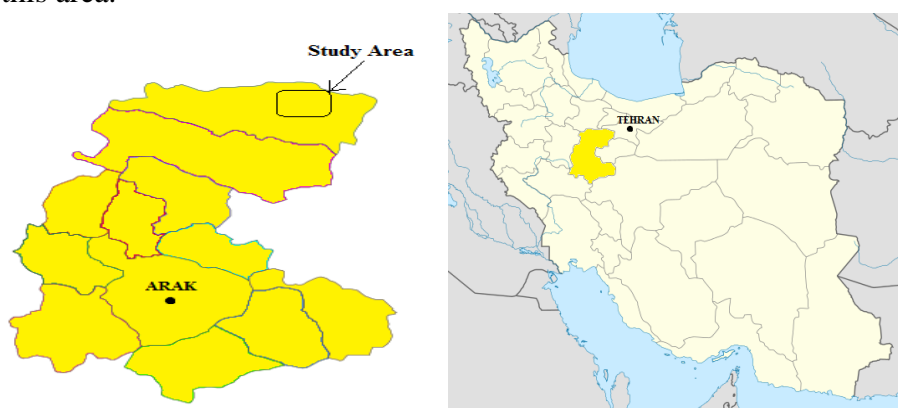
Equation*	R <sup>2</sup>	Author	Rock type
UCS= $9.97e^{(0.02^p \cdot R)}$	R <sup>2</sup> =0.94	Deer and Miller (1966) [2]	different lithologies
UCS= <b>0.4R-3.6</b>	R <sup>2</sup> =0.94	Shorey et al. (1984) [17]	coal
UCS= <b>4.92R-67.52</b>	R <sup>2</sup> =0.93	Sachpazis (1990) [18]	carbonates rocks
UCS= $2.21e^{(0.07 \cdot R)}$	R <sup>2</sup> =0.94	katz et al. (2000) [3]	limestone and sandstone
UCS= $e^{0.059R+0.818}$	R <sup>2</sup> =0.98	Yilmaz and sendir (2002) [19]	Gypsum
UCS= <b>2.75R-36.83</b>	R <sup>2</sup> =0.97	Dincer et al. (2004) [4]	basalts and tuffs
UCS= <b>0.000004R<sup>4.29</sup></b>	R <sup>2</sup> = 0.89	Yaser and Erdogan(2004) [20]	carbonates, limestone
UCS= $1.45e^{(0.07R)}$	R <sup>2</sup> =0.92	Aydin and Basu (2005) [21]	granitic rocks
UCS= <b>3.2R-46.59</b>	R <sup>2</sup> = 0.76	Shalabi et al. (2007) [22]	dolomite, limestone
UCS= <b>0.0028R<sup>2.584</sup></b>	R <sup>2</sup> =0.92	Yagiz (2009) [23]	travertine, limestone, schist
UCS= <b>0.0465R<sup>2</sup>-0.1756I<sub>s</sub>+27.682</b>	---	Torabi et al. (2010) [5]	siltstone, sandstone, shale
UCS= <b>2.262R-29.38</b>	R <sup>2</sup> =0.91	Tondon and Gupta (2014) [6]	Quartzite
UCS= <b>2.73R – 41.78</b>	R <sup>2</sup> =0.96	Tondon and Gupta (2014) [6]	Granite
UCS= <b>1.233R – 2.846</b>	R <sup>2</sup> =0.89	Tondon and Gupta (2014) [6]	Dolomite
UCS= <b>35.54V-55</b>	R <sup>2</sup> =0.64	Tugrul and Zarif (1999) [7]	Limestone
UCS= <b>9.95V<sup>1.21</sup></b>	R <sup>2</sup> =0.69	Kahraman (2001) [24]	dolomite, limestone, marl

Equation*	R <sup>2</sup>	Author	Rock type
UCS= (V <sub>P</sub> -2.0195)/0.032	R <sup>2</sup> =0.81	Yesar and Erdogan (2004) [20]	lime, marble, dolomite
UCS= 22.032V <sup>1.243</sup>	R <sup>2</sup> =0.72	Sousa et al. (2005) [25]	Granites
UCS= 56.71V-192.93	R <sup>2</sup> =0.81	Cobanglu and Beran (2007) [14]	limestone, sandstone
UCS= 62.4V-117.99	R <sup>2</sup> =0.9	Sharma and Singh (2008) [8]	soft rocks
UCS= 133.3V-227.19	R <sup>2</sup> =0.96	Khandewal and Singh (2009) [9]	coal, shale, sandstone
UCS= 0.14V-899.23	R <sup>2</sup> =0.9	Diamantis et al. (2011) [11]	Peridotite
UCS= 0.258V <sup>3.543</sup>	R <sup>2</sup> =0.92	Yagiz (2011) [10]	travertine, soft lime
UCS= 0.005V	R <sup>2</sup> =0.94	Minaeian and Ahangari (2013) [12]	Conglomerate
UCS=0.443e <sup>1.091V</sup>	R <sup>2</sup> =0.84	Pappalardo et al. (2016) [36]	Limestone
UCS= 20.71I <sub>S50</sub> +29.6	---	Deer and Miller (1966) [2]	limestone, granite, basalt
UCS= 16I <sub>S50</sub>	---	Read et al. (1980) [26]	Sedimentary
UCS= 20I <sub>S50</sub>	---	Read et al. (1980) [26]	Basalt
UCS= 23I <sub>S50</sub> +13	---	Cargil and Shakoor (1990) [27]	sandstone, limestone
UCS= 8.41I <sub>S50</sub> +9.51	R <sup>2</sup> =0.85	Kahraman (2001) [24]	dolomite, limestone
UCS= 13I <sub>S50</sub>	R <sup>2</sup> =0.7	Sabatakakis et al. (2009) [13]	I <sub>S</sub> >2MPa
UCS= 24I <sub>S50</sub>	R <sup>2</sup> =0.6	Sabatakakis et al. (2009) [13]	I <sub>S</sub> =2-5MPa
UCS= 28I <sub>S50</sub>	R <sup>2</sup> =0.72	Sabatakakis et al (2009) [13]	Limestone, sandstone
UCS= 22.8I <sub>S50</sub>	R <sup>2</sup> =0.99	Singh et al. (2012) [15]	Schist
UCS= 21.9I <sub>S50</sub>	R <sup>2</sup> =0.89	Singh et al. (2012) [15]	sandstone
UCS= 21I <sub>S50</sub>	R <sup>2</sup> =0.96	Singh et al. (2012) [15]	epidiorite
UCS= 22.3I <sub>S50</sub>	R <sup>2</sup> =0.68	Singh et al. (2012) [15]	limestone
UCS= 22.7I <sub>S50</sub>	R <sup>2</sup> =0.82	Singh et al. (2012) [15]	dolomite
UCS= 10.99I <sub>S50</sub> +7.042	R <sup>2</sup> =0.92	Heidari et al. (2012) [28]	gypsum; (axial)
UCS= 11.96I <sub>S50</sub> +10.94	R <sup>2</sup> =0.94	Heidari et al. (2012) [28]	gypsum; (diametric)
UCS= 13.29I <sub>S50</sub> +5.251	R <sup>2</sup> =0.9	Heidari et al. (2012) [28]	gypsum; (irregular)
UCS= 19.831I <sub>S50</sub>	---	Li and Wong (2013) [16]	Meta-siltstone
UCS= 21.27I <sub>S50</sub>	---	Li and Wong (2013) [16]	Meta-sand stone
UCS= 5.6I <sub>S50</sub> +4.38	R <sup>2</sup> =0.94	Tondon and Gupta (2014) [6]	granite
UCS= 8.597I <sub>S50</sub> +30.72	R <sup>2</sup> =0.78	Tondon and Gupta (2014) [6]	quartzite
UCS= 10.53I <sub>S50</sub> -7.61	R <sup>2</sup> =0.91	Tondon and Gupta (2014) [6]	dolomite

\* UCS uniaxial compressive strength [MPa]; R Schmidt hammer hardness; V rock shear wave velocity [m/s]; I<sub>S</sub> the point load index [MPa]

## Study area

The study area is located in the north of Saveh, Markazi Province, between the latitudes of  $35^{\circ} 24'$  and  $35^{\circ} 25'$  N and longitude of  $50^{\circ} 51'$  and  $50^{\circ} 21'$  E, (Figure. 1). The study area with regard to geological subdivision is a part of Iran central zone and according to geological maps includes alluvial terraces in low-lying areas and a wide variety of limestone rocks in other areas. Around the study area also exist some pyroclastic rocks, conglomerate, sandstone and marl discretely (Figure 2). Due to the great extent of limestone rocks in the north of Saveh, the presence under-construction projects such as roads, railways and important industrial towns in this region and the importance of the geotechnical parameters, developing empirical relations to estimate strength properties rocks should be considered in this area.



**Figure1. The location of the study area**



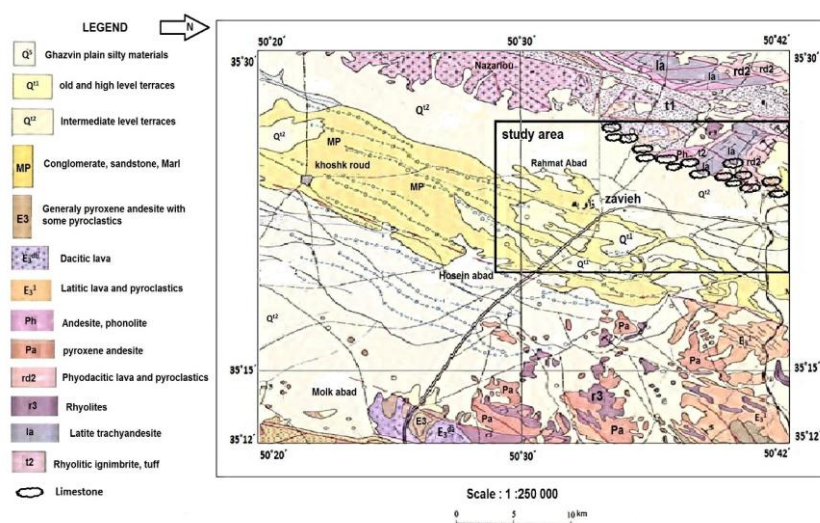


Figure 2. Geological map of the study area and the points sampled [29]

## Data and Method

In this study, due to the lithology and geological similarity, the area is divided to 25 blocks and two samples were taken from each block. After sample preparation, index properties include of bulk density, dry density, water absorption, apparent porosity and water content have been measured. According to the tests results, these indices are  $2.20 \text{ g/cm}^3$ ,  $2.06 \text{ g/cm}^3$ ,  $8.47\%$ ,  $18.2\%$  and  $14.8\%$ , respectively. Then, point load (ASTM D5731) [30], Schmidt hammer (ASTM D5873) [31], wave speed (ASTM D2845) [32], and uniaxial compressive strength tests (ASTM D2938) [1], have been performed on dry cylindrical samples (ISRM 1981) [33].

## Results and Discussion

As mentioned before, point load, Schmidt hammer, shear wave velocity and uniaxial compressive strength tests have been performed on 50 rock samples collected from 25 blocks in the study area. Statistical parameters and the distribution of tests results are given in Table 2 and Figure 3, respectively. According to Table 2 and Figure 3, the average uniaxial compressive strength of rocks in the study area is 37.65 MPa and it varies from 33.5 to 42.6 MPa. The variety of the shear wave velocity values have been achieved 2270 up to 3200 m/s and the average shear wave velocity is equal to 2745 m/s. Figure 3 shows the statistical results of experiments conducted on the samples.

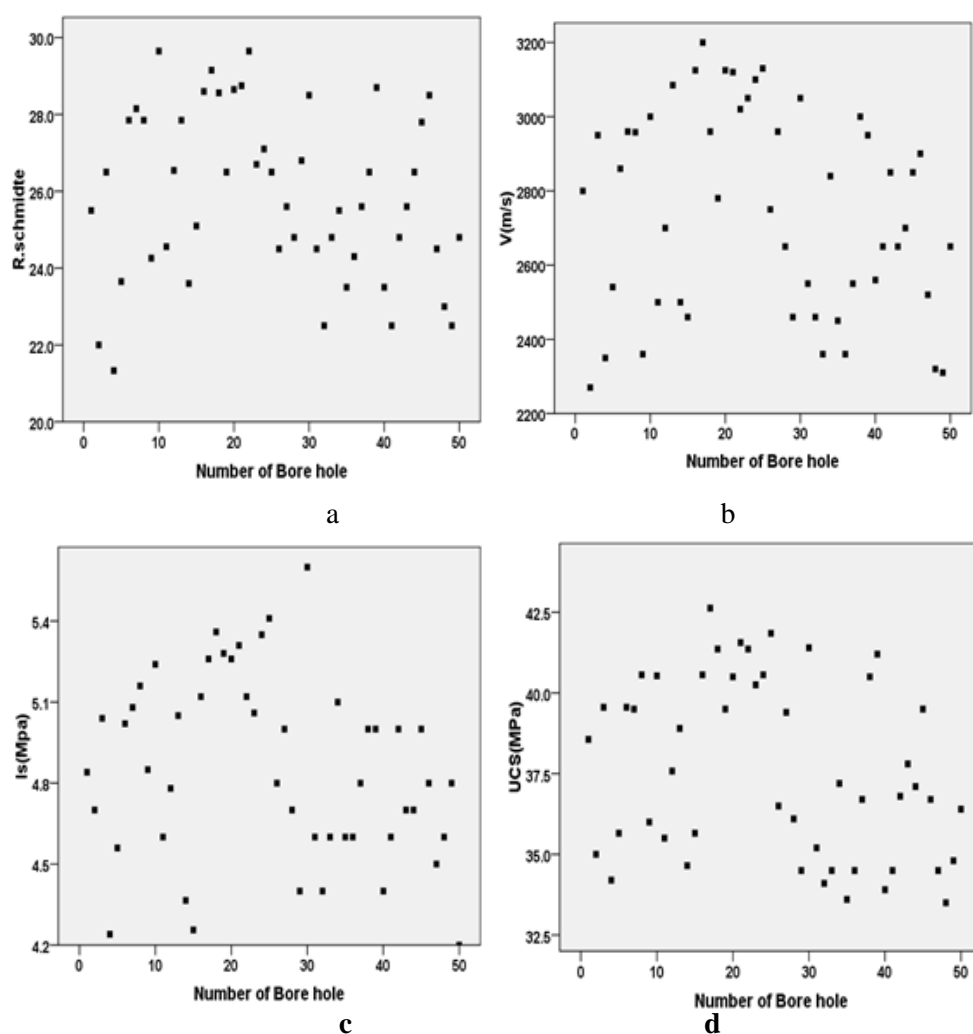
**Table 2. The statistical parameters of the tests results**

Parameters*	UCS [MPa]	V[m/s]	R	Is[MPa]
Number	50	50	50	50
Average	37.65	2745	25.9	4.85
Variance	7.5	7.44E-4	4.8	0.115
Min	33.5	2270	21.3	4.2
Max	42.6	3200	29.6	5.6

\***I<sub>s</sub>**: the point load index; **R**: Schmidt hammer hardness; **V**: Shear wave velocity and **UCS** is the uniaxial compressive strength

Figure 4 also indicates the relation between the uniaxial compressive strength and shear wave velocity, hardness Schmidt, point load index parameters. As can be seen the uniaxial compressive strength increases more than Schmidt hammer hardness, shear wave velocity and the index of pint load increase.

According to the simple regression analysis, the relations (1, 2 and 3) are obtained between the uniaxial compressive strength and shear wave velocity, Schmidt hardness and point load index, respectively.



**Figure 3. a) Distribution of tests results; Shear wave velocity, b) Schmidt hammer, c) uniaxial compressive strength, d) point load index**

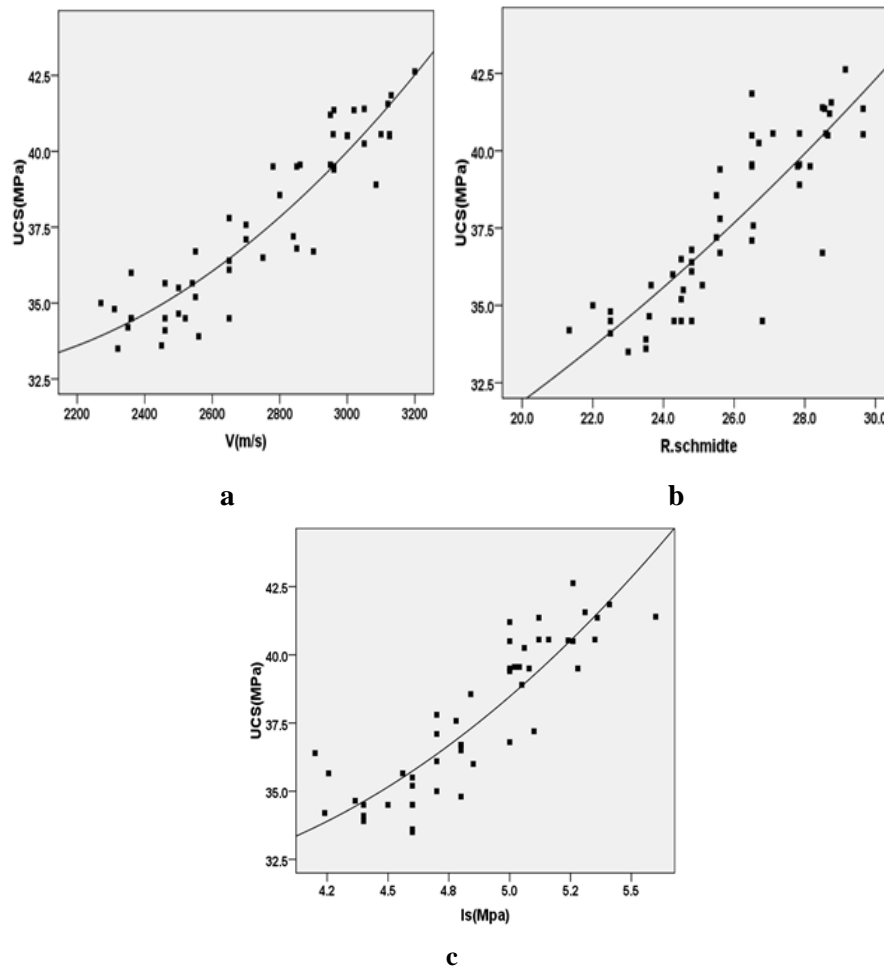
$$UCS = -4.6 \times 10^{-6} V^2 - 0.0164V + 46.7 \quad R^2 = 0.86 \quad (1)$$

$$UCS = 0.02R^2 + 0.047R + 23 \quad R^2 = 0.77 \quad (2)$$

$$UCS = 2.115I_s^2 - 13.46I_{s_{50}} + 52.9 \quad R^2 = 0.78 \quad (3)$$

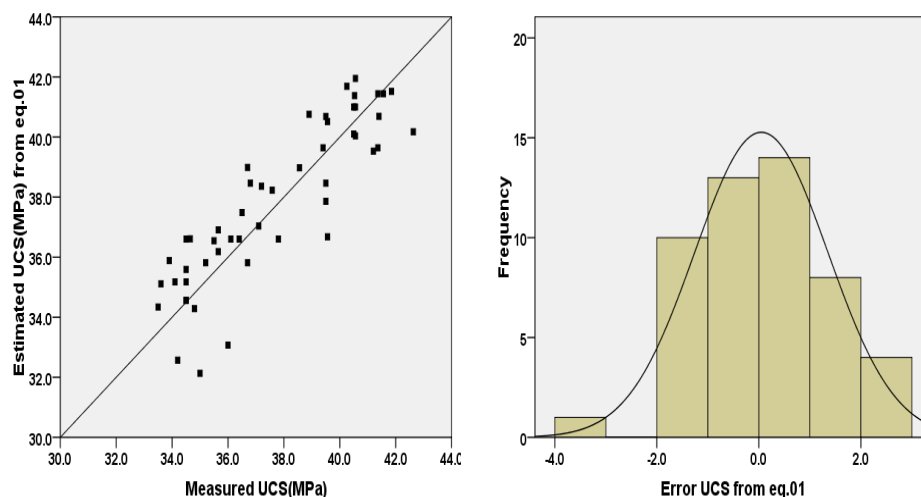
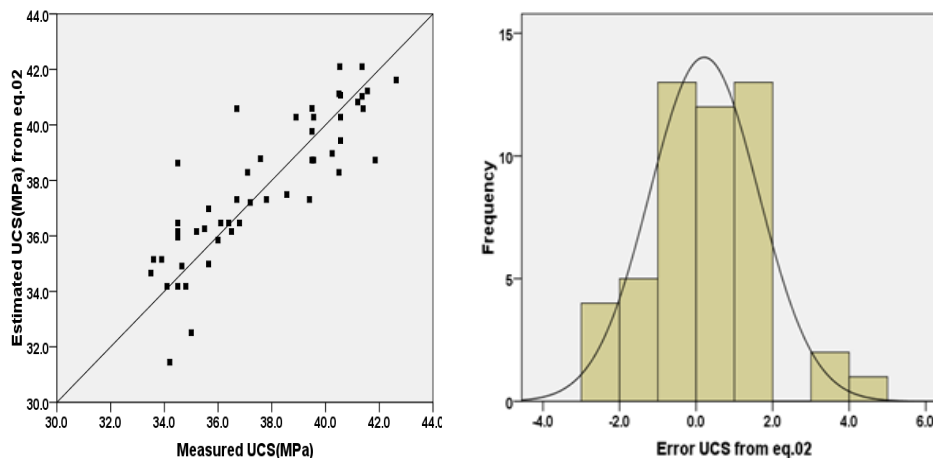
where UCS and  $I_{S50}$  are the uniaxial compressive strength and the index value of point load, respectively,  $V$  is the shear wave velocity in m/s,  $R$  is Schmidt hardness value.

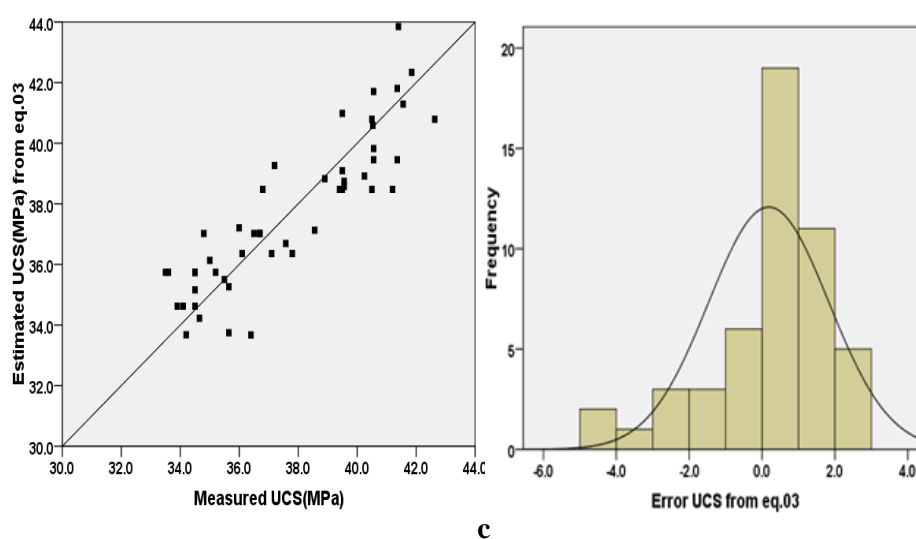
To investigate the accuracy of the relations, the values of the compressive strength of the relations have been compared with the



**Figure 4. a) The relation between UCS and the Schmidt hardness, b) Shear wave velocity, c) and the point load index**

measured values from direct compressive strength tests results (Figure. 5). Since frequency distribution of errors is coincident with normal curve, reliability of the relations can be inferred. According to the results (Figure. 4), the proposed relations have high reliability to estimate the uniaxial compressive strength and practical in geotechnical engineering designs.

**a****b**



**Figure. 5. The relation between the uniaxial compressive strength using direct test and the proposed relations and their statistical error, a) relation 1, b) relation 2, c) and relation 3**

In this study, using Nash empirical coefficients ( $E$ ) and root mean square error (RMSE), the accuracy of empirical derived relations is studied. Nash coefficient ( $E$ ) applies to evaluate the prediction ability of relations and is defined in the range of one to negative infinity as explained in Equation 4. Root mean square error or RMSE is calculated in accordance with Equation 5.

$$E = 1 - \frac{\sum_{i=1}^n (N_{i(0)} - N_{i(P)})^2}{\sum_{i=1}^n (N_{i(0)} - N_{i(P)})^2} \quad (4)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (N_{i(0)} - N_{i(P)})^2}{n}} \quad (5)$$

where  $N_{i(0)}$  and  $N_{i(P)}$  are the observed and predicted values respectively, and  $n$  is the number of data. As the calculated  $E$  and  $RMSE$  values be closer to one and zero, respectively, the relation performance would be more appropriate. The calculated values for three relations have been showed in Table 3.

**Table 3. The  $R^2$ ,  $E$  and  $RMSE$  values related to proposed relations**

Relations	$R^2$	$E$	$RMSE$
I	0.86	0.753	1.344
II	0.77	0.723	1.426
III	0.78	0.771	1.296

According to Table 3, with regard to  $R^2$ ,  $E$  and  $RMSE$  factors, relation III (e.g. Equation 3) is more accurate and reliable. In addition, proposed relations have been compared with other researchers' relations. So, Equations 1, 2 and 3 have been compared with the Pappalardo (2016), Shalabi (2007) and Tondon (2014) relations, respectively (relations 6-8).

$$\text{Pappalardo (2016)} \quad UCS = 0.443e^{1.09V} \quad R^2 = 0.84 \quad (6)$$

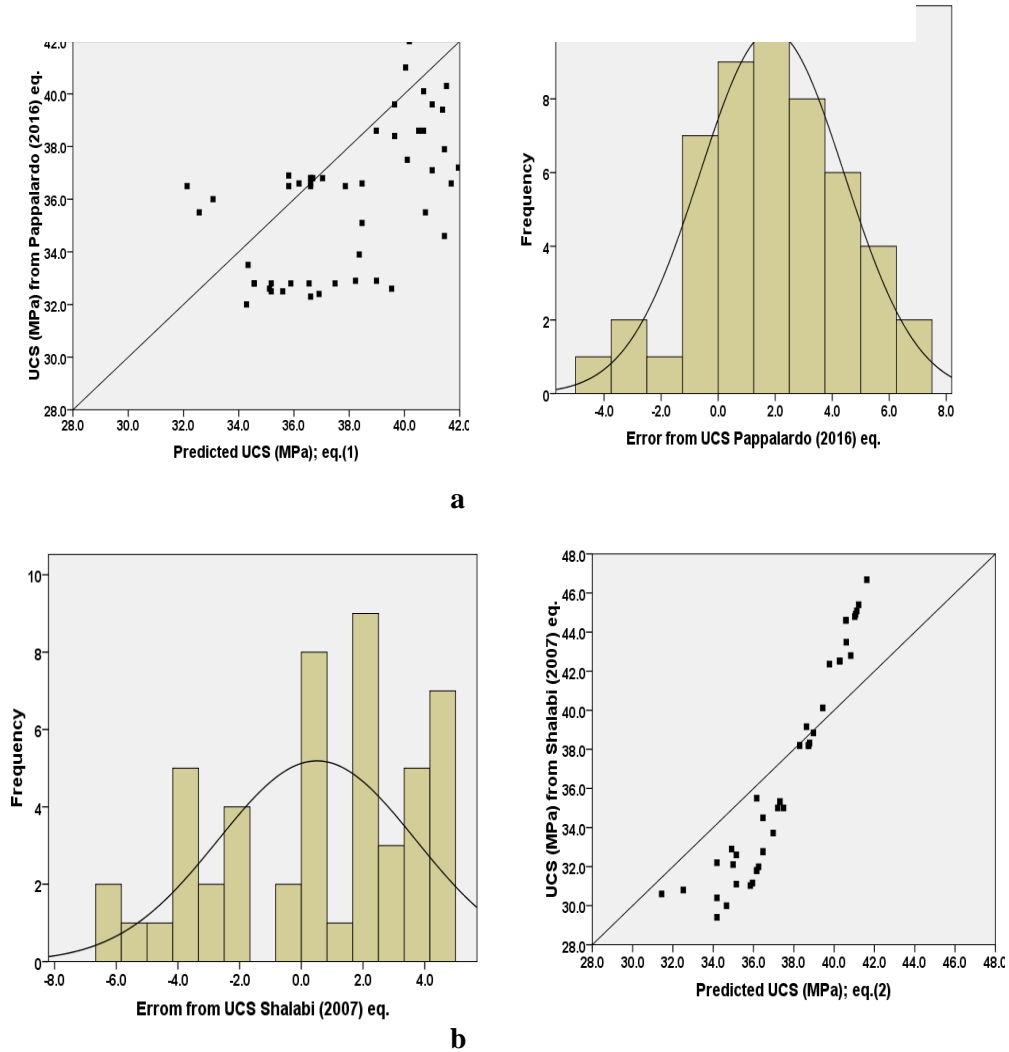
$$\text{Shalabi (2007)} \quad UCS = 3.7R - 46.6 \quad R^2 = 0.76 \quad (7)$$

$$\text{Tondon (2014)} \quad UCS = 5.6I_{S_{50}} + 4.38 \quad R^2 = 0.94 \quad (8)$$

where  $UCS$  and  $I_{S_{50}}$  are the uniaxial compressive strength and the index value of point load, respectively,  $V$  is the shear wave velocity in m/s,  $R$  is Schmidt hardness value.

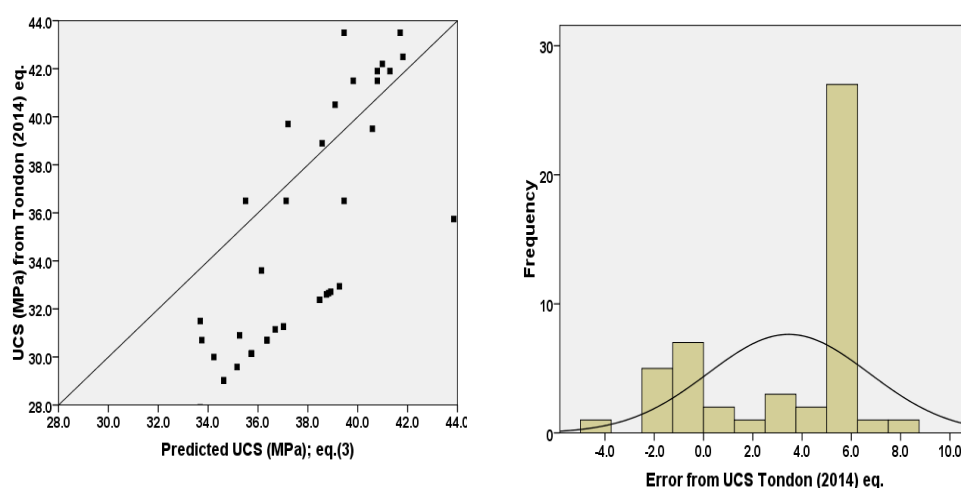
As shown in Figure 6, these relations generally have slight differences to proposed relations in  $UCS$  value. Median difference value between these relations and Shalabi (2007), Tendon (2014) and

Pappalardo (2016) relations is 1.89, 3.45 and 0.505, respectively. However, these relations with reliable accuracy can be used as an indirect suitable estimation for USC.



**Figure. 6. a) Comparison of the UCS from proposed relations with Pappalardo (2016), b) Shalabi (2007),**





c

Figure. 6. c) Tondon (2014)

## Conclusion

Considering the projects development especially in large cities, extensive knowledge of geotechnical parameters including uniaxial compressive strength of rocks is necessary. In these conditions, development of the relations to determine the uniaxial compressive strength of rocks with indirect method can reduce the cost and increase speed of the strength properties estimation. In this study, according to significant presence of limestone in the area around Saveh, several tests including shear wave velocity, point load index, Schmidt hammer and uniaxial compression strength on 50 samples of limestone have been performed. Then the relations to estimate uniaxial compressive strength of limestones tests have been presented. Finally, to evaluate proposed relations, they have been compared with

other researchers (Pappalardo, 2016, Shalabi, 2007 and Tandon, 2014). The result showed that Equations (1) and (3) have the same values of UCS with Shalabi (2007) and Tandon (2014) relations, respectively. The results indicate that the proposed relations can offer a good estimation of uniaxial compressive strength of limestones with acceptable approximation in the study area. The proposed empirical relations are applicable for limestones in study area or rocks with similar characteristics in other places.

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