Explicit solution of the three-layer soil period, based on the theory of wave vibration equations

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Received: 2022/8/2 Accepted: 2022/5/1

Abstract

The soil's natural period has been simulated in more realistic conditions in recent years using complex equations. In previous studies, the natural period of soils made of bilayer soils with different characteristics has been determined and formulas have been presented. For three-layer soils, the proposed relations are complex and performed by approximate methods. Those methods are inappropriate to fast and precise calculation. In this study, the natural period of the soil has been solved using explicit and closed solution of wave propagation method. The final equations are simplified to utilize the advantages of this method are providing simple relations and high accuracy of answers. Likewise, the natural period of two-layer soil is determined based on the proposed method. the natural ground period of the layers around Karaj alluvium has been determined using the above method. Additionally, considering a few ordinary conditions (depth of layers and soil qualities), the natural period of the soil has been determined and introduced by graphs. As indicated by the outcomes, weak layer cause to increase the ground natural period, while stiff layer decreases the period. The rate of ground period sharply reduces by considering the strong soil in bottom of the layers. In general, increasing the layers specification ratio, reduce ground natural period; excluding the case where the order of the layers is strong to weak. The difference between the values of period in both states are significant.

Keywords: natural period of the soil, soil profile, wave vibration equations, analytical solution, explicit method

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1.Introduction

The spread of science has demonstrated that the harm brought about by earthquake is straightforwardly related with the geological condition of the ground. During the earthquake, seismic waves spread from heterogeneous soil residue that are on the rough bed, prompting changes in the wave qualities. This phenomenon is due to the amplification or attenuation of the wave amplitude in addition to the change in frequency characteristics. This peculiarity principally relies upon modal characteristics of the soil sediments that are situated on the bed rock. It should be noted that the phenomenon of wave amplification is predominant in the vicinity of the basic frequency of soil deposition. A few examinations on the geotechnical investigation of earthquakes [1, 2] show that the intensity of damage to structures due to earthquakes depends on the seismic characteristics of underground sediments (such as depth of layers, stiffness, and their reinforcement characteristics). This study accurately calculates the amount of natural period of multilayer soils by explicit method (analytical solution).

Sedimentary soils can improve the seismic qualities of earthquakes. These sorts of changes are typically as an expansion in greatest speed increase as well as length. Obviously, it ought to be noticed that most urban communities are additionally based on sedimentary layers. Subsequently, it will be vital to concentrate on the intensification phenomenon, which is as a rule because of the expansion in ground movement because of the presence of soil dregs.

Vibration mode shape and modular participation coefficient are among the essential data expected to understand the dynamic response of each structure and to evaluate the functional accuracy of the results. PC programming for example, SHAKE and SASSI are utilized to address soil model reactions in the frequency domain, however these codes can't compute the frequency and shape of the mode.

The location of structure is delegated follows. For stiff soils, parameters A and B, and by diminishing the hardness of the soil (soft soils), parameter E obtained, individually. Trial information and insightful examinations have shown that materials near the ground surface greatest effect on the movement of ground. The period of the soil likewise relies upon the kind and depth of the layer. To this end the normal soil period frame is introduced in Figure (1).

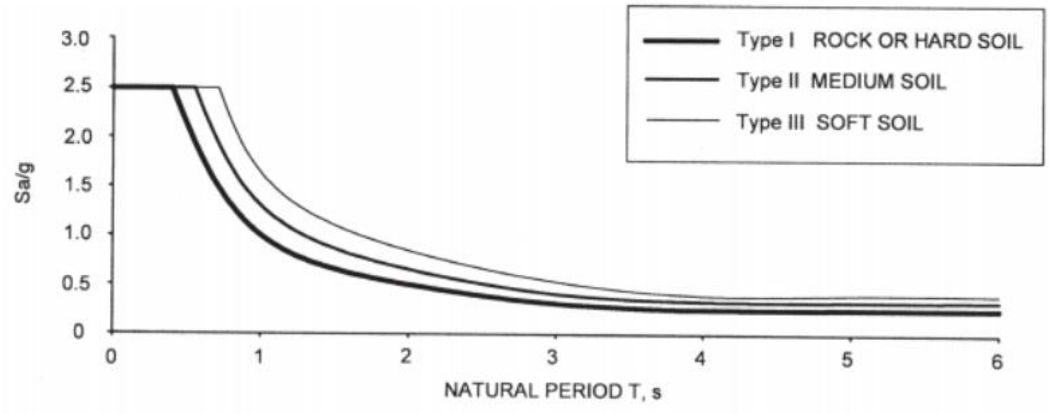


Figure 1. Spectral response of natural soil based on vibration period [3]

In this way, one of the significant parameters for seismic assessment is the natural period of soil. One basic scientific method to assess the seismic impact of natural soil period located on the bed rock is to consider the heterogeneous silt layer as a homogeneous one-layered (1D) that has a similar shear wave velocity. In examinations, it has been shown that the two-layered period of natural soil is practically corresponding to the essential one-layered period [4, 5]. Most guidelines give the typical shear wave velocity on 30 m profound soil silt as a characterization of natural soil parameters [6, 7]. Then again, a few guidelines, like Japan, consider the natural period of soil residue as a seismic order of natural soil [8]. In such manner, a few related examinations show that the natural period of the soil layer on the bed rock is a more reasonable parameter to foresee the resonance ratio of the typical shear wave velocity of the 30-meter layer, particularly for layers with extensive stretches [9]. Consequently, the natural period of the soil is utilized as the fundamental parameter to work out the hardness and depth of soil residue over the bedrock. In this study, the point is to calculate the natural period of multi-layer soil by analytical method (explicit solution) so the parameters of natural soil are determined all the more precisely and resulting breaks down are determined all the more precisely. At long last, the plan of designing designs will be finished with a higher unwavering quality. Different methods have been proposed to work out the natural period of natural soil (single-layer or multi-facet). Meanwhile, Vijayendra and Nayak, (2014) [10] have introduced a worked-on method for the examination of layered sedimentary soils situated on the bed rock. This method depends on two- parameter equation relapse. Dobry et al. 1976 [11] offers a worked-on method for assessing soil layer period. Vijayendra et al.

2010 [12] An examination of the two methods for mean shear wave velocity and the Madras method for working out the natural period of the soil and the model of a few levels of discrete opportunity were performed and contrasted and genuine earthquakes information. Larkin and Houtte 2004 [13] led a study to decide the period of the natural soil. The above study depends on NZS1170.5 strategy. This method is based on several methods of determining the characteristics of the soil. Kwok et al. 2007 [14] for nonlinear seismic reaction of the soil, they have utilized the arrangement of the wave propagation package. Hadijan 2002 [15] has likewise concentrated on the natural soil period and the vibrational methods of soil layers. As per the researcher, the method introduced in this study without the requirement for reiteration (in simplifying solutions), for example, Dobry et al. 1976 [11] and Madera 1970 [16]. The method for these two researchers depends on the worked-on Riley wave method, which requires iterative tests and introductory assessment of the mode shape. The Madera method is determined for two-layer frameworks, as a rule utilizing diagrams. In this study, the researcher has substituted an approximate method for implementation in the spreadsheet. The seismic response of viscoelastic layered soils has been investigated analytically by Sarma 1994 [17]. In this study, the movement on the soil is flat and both consistent and transient states have been researched. Zhao 1966 [18] assessed the modal parameters for a simple soil whose shear wave velocity dispersion is linear. Additionally, in 1967 [19] the modal analysis of soft soils, which includes radiative damping, was investigated.

Konno and Ohmachi 1998 [20] determined the motion characteristic of the soil utilizing the spectral ratio between the horizontal and vertical components of the microtremor. Tsai and Housner 1970 [21] have gotten semi-infinite surface movement of the soil. Vibrational characteristics of soil sediments have also been investigated using different wave velocities (Gazetas 1982 [22]). Rovithis et al. 2011 [23] have investigated the harmonic response of heterogeneous layered soils with an analytical approach. In this study, soil behavior was considered as one-layered viscoelastic. It is additionally expected that the layer is heterogeneous, which is put on the soil layer with high hardness and homogeneity. The above logical method is acquired by explicit solution of Bessel equation for natural frequency. The proposed model is approved utilizing the finite element numerical method.

Wkamatsu and Yasui 1996 [24] investigated the possibility of estimating the intensification of sedimentary soil characteristics based on the horizontal to vertical ratio of spectral microtremors. Park and Hashash 2004 [25] obtained multilayer soil damping using nonlinear temporal analysis response. Earthquake period in Karaj alluvium and intensification ratio were determined (Ghanbari et al. 2010 [26]).

As referenced, the natural period of the soil is utilized as a critical parameter to compute the hardness and depth of the soil layer over the bed rock. Soil alluvium in Karaj is used as a case study. Because of its topographical area, this city is situated on dynamic faults like the North of Tehran and Mosha. This issue can likewise produce seismic earthquakes of more than 7 Richter. As indicated by this, definite arrangement assists with finding the exact ground layer period, which will lead to more optimum analyzes. For better comprehension of the contrast between a single-layer and a multi-layer ground period, refer to Figure 2.

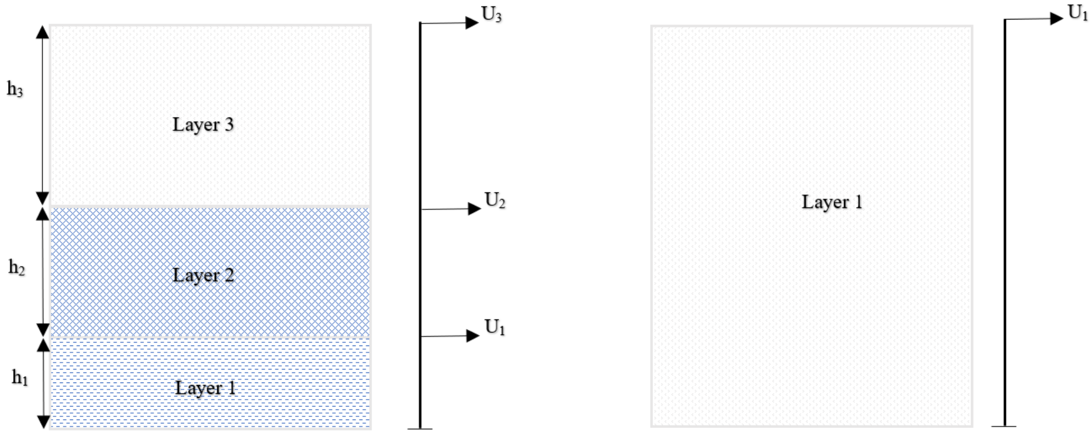


Figure 2. schematic period of multi-layer soil and an equivalent layer

1.1 Methodology and solving equations of displacement of the soil

To comprehend the method for solving the equation of movement and applying boundary conditions, first the equations of movement related with bilayer soil are solved and afterward the principal soil parameters Multiple layers are extended and made sense of.

1.2 Parametric solution of monolayer soil

Assume the displacement equation of one-layer soil profiles as Equation (1).

$u(z,t) = Ae^{i(\omega t+kz)} + Be^{i(\omega t-kz)}$	(1)

Which, parameter ω demonstrate the rotational frequency of the quake, k is the wave number and is determined utilizing the equation ω/v_s. parameters A and B are the amplitude of the wave in the negative and positive direction of the z-axis, separately. At soil surface, the value of shear pressure and shear strain should be equivalent to nothing, thus:

$$\tau(0,t) = G\gamma(0,t) = G\frac{\partial u(0,t)}{\partial z} = 0 \tag{2}$$

With replacement equation 1 in Equation 2, the outcome is:

$$Gik\left(Ae^{ik(0)} - Be^{-ik(0)}\right)e^{i\omega t} = Gik(A-B)e^{i\omega t} = 0 \tag{3}$$

Accordingly, by solving the equation 3, constants parameters of Equation (1) are gotten as follows:

A=B

Accordingly, the relocations of layer will be (equation 4):

$$u(z,t) = 2A\frac{(e^{ikz}+e^{-ikz})}{2}e^{i\omega t} = 2A\cos kz\, e^{i\omega t} \tag{4}$$

equation (4) is modified as follows to show more sensible way of behaving (soil layer damping,). Thusly, the displacement equation will be as equation (5).

$$u(z,t) = Ae^{i(\omega t+k^*z)} + Be^{i(\omega t-k^*z)} \tag{5}$$

Which, the parameter k* is a complex wave number with the real and imaginary part k1 and k2, separately. By repeating mathematical operations and Simplifications, the displacement function within the presence of damping is likewise applied to the alluvial layer of the bedrock (allude to equation 6).

$$F_2(\omega) = \frac{1}{\cos k^*H} = \frac{1}{\cos\left(\frac{\omega H}{v_s^*}\right)} \tag{6}$$

Considering that the frequency is independent of attenuation. Consequently, the mixed shear model is determined as Equation (7).

$$G^* = G(1+i2\xi) \tag{7}$$

Hence, as indicated by Equation (7), the shear wave velocity is changed as Equation (8).

$$v_s^* = \sqrt{\frac{G^*}{\rho}} = \sqrt{\frac{G(1+i2\xi)}{\rho}} \approx \sqrt{\frac{G}{\rho}}(1+i\xi) = v_s(1+i\xi) \tag{8}$$

For the case that the damping value is small, the wave number will be in the form of relation (9).

$k^* = \frac{\omega}{v_s^*} = \frac{\omega}{v_s(1+i\xi)} \approx \frac{\omega}{v_s}(1 - i\xi) = k(1 - i\xi)$	(9)

Finally, the displacement function is in the form of equation (10)

$F_2(\omega) = \dfrac{1}{\cos k(1 - i\xi)H} = \dfrac{1}{\cos\left[\dfrac{\omega H}{v_s}(1 + i\xi)\right]}$	(10)

Thus, as per the relation $|\cos(x + iy)| = \sqrt{\cos^2 x + \sinh^2 y}$, the resonance function is demonstrated with equation (11)

$\lvert F_2(\omega)\rvert = \dfrac{1}{\sqrt{\cos^2 kH + \sinh^2 \xi kH}}$	(11)

$\sinh^2 y = y^2$ can be considered for small amount of y. equation 11 rewritten as (see equation 12):

$\lvert F_2(\omega)\rvert = \frac{1}{\sqrt{\cos^2 kH + (\xi kH)^2}} = \frac{1}{\sqrt{\cos^2(\frac{\omega H}{v_s}) + \left(\xi\left(\frac{\omega H}{v_s}\right)\right)^2}}$	(12)

Subsequently, the n(th) natural frequency of layered soil will be as Equation 13

$\omega_n = \frac{v_s}{H}\left(\frac{\pi}{2} + n\pi\right), n = 0, 1, 2, \ldots, \infty$	(13)

1.3 Parametric arrangement of two-layers soil displacements

As indicated by the profile of soil, boundary equations can be considered as relations 14 to 16:

1- $\tau(0, t) = 0$	(14)
2- $u_1(h_1, t) = u_2(0, t)$	(15)
3- $\tau_1(h_1, t) = \tau_2(0, t)$	(16)

equations (14) to (16) represents the free surface of zero stress, equal displacement and shear stress at the boundary of layer 1 and 2, respectively. Hence, the displacement equation of the soil is as equation (17) and (18) (it ought to be noticed that in this study, soil layer numbering considered as top to down).

$u_1(z, t) = A_1 e^{i(\omega t + k_1^* z_1)} + B_1 e^{i(\omega t - k_1^* z_1)}$	(17)
$u_2(z, t) = A_2 e^{i(\omega t + k_2^* z_2)} + B_2 e^{i(\omega t - k_2^* z_2)}$	(18)

According to the shear stress condition at soil level, the values of parameters A_1 and B_1 is equal. Additionally, because of the similarity of both layers, following circumstances should be satisfied.

$u_2(z_2 = H) = u_1(z_1 = 0)$	(19)
$\tau_2(z_2 = H) = \tau_1(z_1 = 0)$	(20)

By substituting equations (17) and (18) in the boundary and initial conditions (relations 19 and 20), the outcome will be:

$A_1\left(e^{ik_s^*H} + e^{-ik_s^*H}\right) = (A_2 + B_2)$	(21)
$A_1G_1ik_1^*\left(e^{ik_1^*H} - e^{-ik_1^*H}\right) = G_2ik_2^*(A_2 - B_2)$	(22)

Equation (22) can also be presented as relation (23)

$\frac{G_1k_1^*}{G_2k_2^*}A_1\left(e^{ik_1^*H} - e^{-ik_1^*H}\right) = A_2 - B_2$	(23)

Alpha parameter is utilized to simplify equation (23)

$\frac{G_1k_1^*}{G_2k_2^*} = \frac{\rho_1v_{s1}^*}{\rho_2v_{s2}^*} = \alpha_z^*$	(24)

v_{s2}^* v_{s1}^* are shear wave velocity in the second and first layer respectively. α_z^* is the complex impedance ratio. By coupled solving of equations (19) and (20), the constants parameters of the displacement are obtained.

$A_2 = \frac{1}{2}A_1[(1+\alpha_z^*)e^{ik_1^*H} + (1-\alpha_z^*)e^{-ik_1^*H}$	(25)
$B_2 = \frac{1}{2}A_1[(1-\alpha_z^*)e^{ik_1^*H} + (1+\alpha_z^*)e^{-ik_1^*H}$	(26)

assumed that, the vertical propagation of the intensified shear wave, demonstrated by parameter A, move upwards along the first layer. If the second layer does not exist, the effect of the free surface will be reflected in the first layer and cause to produce an intensified motion equal with 2A at the bedrock (beginning of the first layer). Within the sight of the subsequent layer, the value of intensification displacement in the free surface is determined as Equation (27)

$2A_1 = \frac{4A}{(1+\alpha_z^*)e^{ik_1^*H}+(1-\alpha_z^*)e^{-ik_1^*H}}$	(27)

As per the definition, the overall relative displacement ratio is obtained as equation (28)

$F_3(\omega) = \frac{u_1}{u_2} = \infty$ $F_3(\omega) = \frac{2}{(1+\alpha_z^*)e^{ik_1^*H}+(1-\alpha_z^*)e^{-ik_1^*H}}$	(28)

Which is modified using O'Leary law (equation 29)

$F_3(\omega) = \frac{1}{\cos\left(\frac{\omega H}{v_{S1}^*}\right)+i\alpha_z^* \sin\left(\frac{\omega H}{v_{S1}^*}\right)}$	(29)

To calculate the natural period of the soil, the values of the parameters k are calculated; in the next step, parameter ω is obtained. By knowing the ω and k parameters the value of T (natural period of the ground) is calculated. The calculation process is described as follows.

Also, in process of solving and expanding (equation 27), eventually the alpha value will be equal to relation (30).

$\alpha = \cot(k_1 h_1) \times \cot(k_2 h_2)$	(30)

Where the parameter α is equivalent to $\frac{\rho_1 v_{S1}}{\rho_2 v_{S2}} = \frac{G_1 k_1}{G_2 k_2}$. By substituting equation (31) and alpha equation in equation (30) ω will be obtained as:

$k = \frac{\omega}{v_s}, \quad k_1 = \frac{\omega}{v_{s1}}, k_2 = \frac{\omega}{v_{s2}}$	

At long last, by indicating the parameter ω, the natural period of the soil will be gotten (Equation 31):

$\omega = \frac{2\pi}{T} \quad (\omega = constant)$	(31)

To compute the natural period of the soil, can be accepted that the ratio of $F_3(\omega) = \frac{u_1}{u_2} = \infty$ is the displacement of the soil surface to the bedrock, this infinite value represents the maximum possible displacement. By substituting the parameters in above ratio, the fixed parameters (A and B) are eliminated. The fixed values of k, ω, v_s, G and ρ were indicated; and afterward, the parameter k will figure out which will lead to the determination of ω. Eventually, the period of the natural soil can be determined utilizing the described equations.

1.4 Parametric solution of three-layer soil displacements

According to the new profile of the soil, new boundary conditions of the second layer are added as follow:

$u_2(h_1, t) = u_3(0, t)$	

So:

$A_3 + B_3 = A_2 e^{ik_2 h_2} + B_2 e^{-ik_2 h_2}$	(32)

Furthermore,

$\tau_2(h_1,t)=\tau_3(0,t)$	
$G_2k_2(A_2e^{k_2h_2}-B_2e^{k_2h_2})=G_3k_3(A_3-B_3)$	
$\mathrm{A_3-B_3=\alpha_2\left(A_2e^{ik_2h_2}-B_2e^{-ik_2h_2}\right)}$	(33)
$\mathrm{A_3=\frac{1}{2}\left[(1+\alpha_2)A_2e^{ik_2h_2}+(1-\alpha_2)B_2e^{-ik_2h_2}\right]}$	(34)
$\mathrm{B_3=\frac{1}{2}\left[(1-\alpha_2)A_2e^{ik_2h_2}+(1+\alpha_2)B_2e^{-ik_2h_2}\right]}$	(35)

Same as previous section, the obtained constant parameters are substitute in the displacement ratio; then by solving and simplifying the obtained equation, the alpha value is determined (equation 36) and furthermore the value of ω is gotten with equation (37).

$\alpha=\tan\alpha\cot c-\alpha_1\tan b\cot c-\alpha_2\cot a\tan b$	(36)
$\cos a\cos b\cos c-\alpha_1\sin a\sin b\cos c-\alpha_2\cos a\sin b\sin c-$ $\alpha\sin a\cos b\sin c=0$	(37)

a, b and c parameters in the alpha equation are characterized as follows. Likewise, α1 and α2 are:

$\alpha=\mathrm{k}_1h_1,\quad b=k_2h_2,\quad c=k_3h_3$	
$\alpha_1=\frac{\rho_1v_{S1}}{\rho_2v_{S2}}$	
$\alpha_3=\frac{\rho_2v_{S2}}{\rho_3v_{S3}}$	

Alpha is obtained by the multiple of α1 and α3. ω is determined same as the previous section.

$\alpha=\alpha_1\times\alpha_3$	

Where

$T=\frac{2\pi}{\omega}\quad(\omega=constant)$	

The one-layer formula, proven in this method, in addition to being an explicit method for calculating the natural period of the ground, is also very simple and practical. Studies have additionally been performed on the natural period of the bilayer soil, but no explicit relations have been provided for the three layers soil. Since the equations become more complex as the layers number increases, it turns out to be undeniably challenging to introduce equations. In this study, the relations governing the three layers are formulated in a simple and practical way. Therefore, relations are given that are simple to utilize. One more benefit of this method is that, assuming the layers are put on top of one another, many changes in the natural period of the soil should be visible in three layers, and subsequently, the presentation of the designs is better anticipated and dissected and planned with high precision. The displacement of the layers can be understood using the simplified relation. Also, calculation time will be shorter.

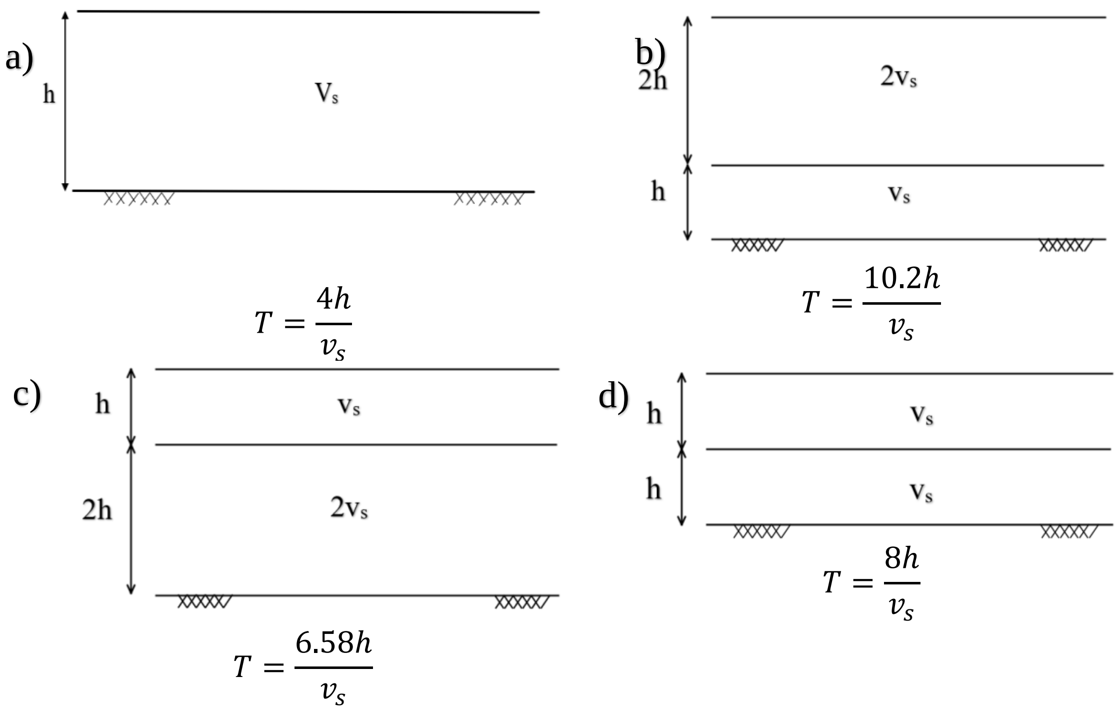
For n-soil layer, the above equations can be determined, yet because of the intricacy of the relations, it is not possible to reach a simple explicit solution and no more than three layers have been discussed. Three-layer and two-layer modes are more tangible in real projects.

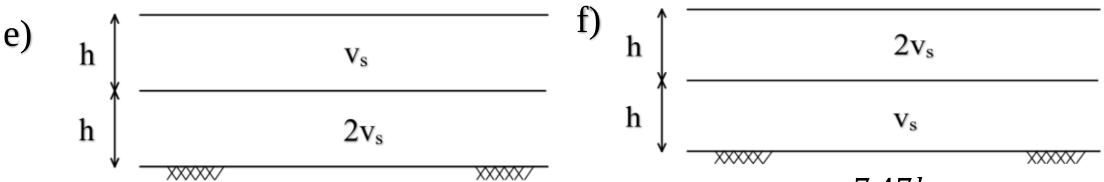
2.Results

The following sections present period of two- and three-layered soil in different specifications (v_s) and depths (h). To better classification, it is assumed that the ratio of each two layers remain constant.

2.1. Two-layer ground

Figure (3) shows the two layered soil natural period in various soil details and depths. In the following figures (b and c), depth and shear wave velocity ratio are 2 and 0.5. Increasing the soil specification ration, increases the natural period of the ground.





$$T = \frac{5.1h}{v_s}$$

$$T = \frac{7.47h}{v_s}$$

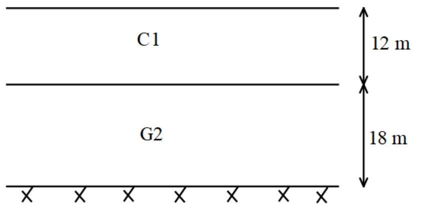
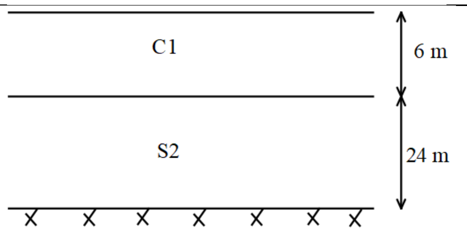
Figure (3). The natural period of two-layer ground for different stiffness ratio

It can be concluded that, stiff soil under the soft layer reduces the natural soil period. In any case, this condition can be weakened by changing the layers determinations.

2.2. Case study of Karaj alluvium

According to the data obtained from different regions of Alborz province, the values of natural periods of the soil in light of the introduced equations are displayed in table (1).

Table (1). Karaj soil alluviums specifications and period, calculated by proposed method

Karaj alluviums	Soil characterization	Soil period	Soil profile
Rajai Shahr (Gohardasht)	$\rho_{G1} = 16.7, v_{sG1} = 208$ $\rho_{G2} = 18.6, v_{sG2} = 330$	T1=0.16 T2=0.38	
Hesarak	$\rho_{s1} = 17.5, v_{ss1} = 233$ $\rho_{s2} = 18.5, v_{ss2} = 309$	T1=0.156 T2=0.414	
Golshahr	$\rho_{s1} = 16.2, v_{ss1} = 209$ $\rho_{s2} = 16.8, v_{ss2} = 277$	T1=0.136 T2=0.384	

Jahanshahr	$\rho_{s1} = 15.8 , v_{ss1} = 189$ $\rho_{s2} = 17.1 , v_{ss2} = 251$	T1=0.122 T2=0.363	

2.3. three-layer ground

For a specific case of a three-layer soil profile (Figure 4), the natural soil period frame is obtained as equation 35.

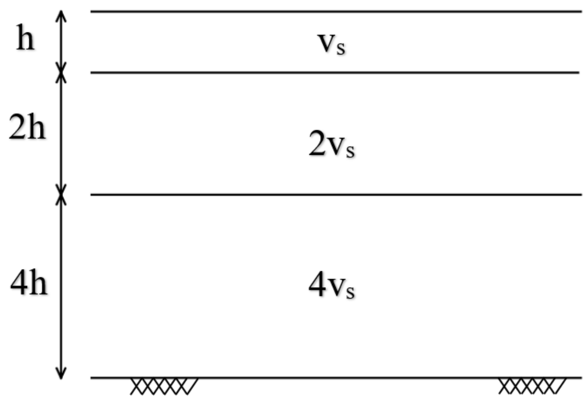


Figure 4. Three-layer natural soil profile with a stiffness ratio of 2

$T = \frac{8.217h}{v_s}$	(38)

It ought to be noticed that equation 36 is the simplified and final period of soil profile. For all potential cases the above equation is solved and displayed in diagrams in Figure (5). For instance, if the order of the layers from top to bottom is strong to weak (it is considered a strong layer that has a higher stiffness than other layers and the weak layer will have the lowest stiffness), graph (a) will be used. In the above diagram, assuming the stiffness ratio of the layers in pair is approximately equivalent to 2, the natural period of the soil is determined to be 20.08. This value is equivalent to 2.2464 if the order of layers is strong, weak, and medium, respectively.

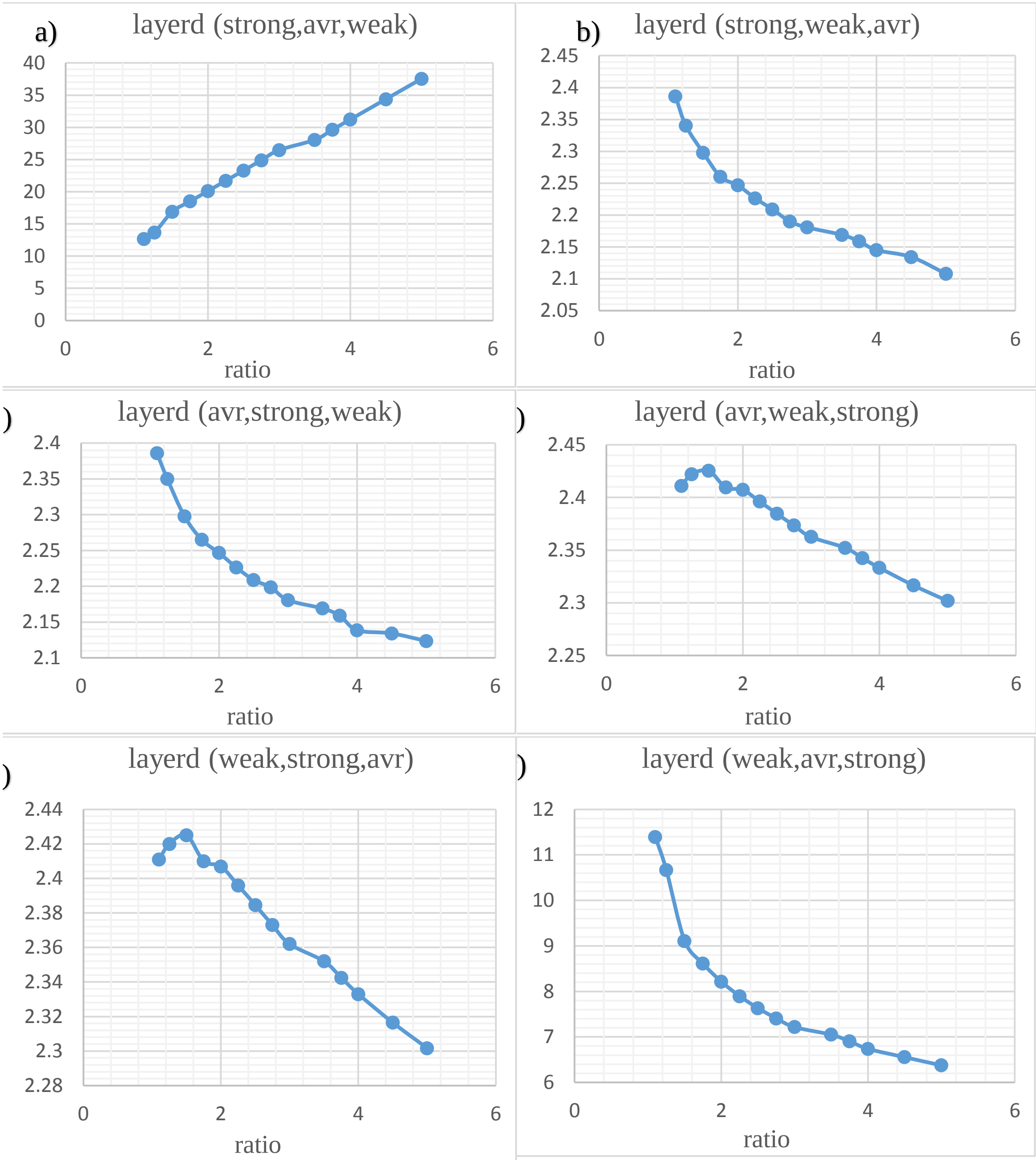


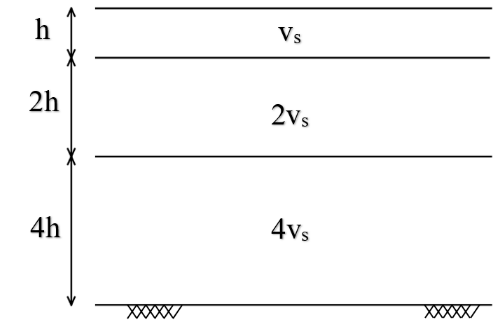
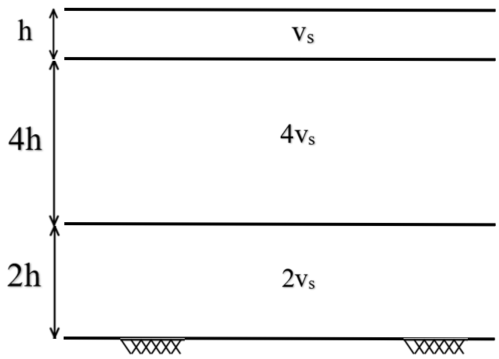
Figure 5. Three-layer natural ground period with different layer arrangement and different stiffness ratios

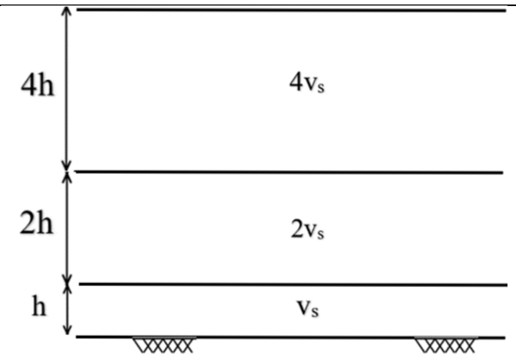
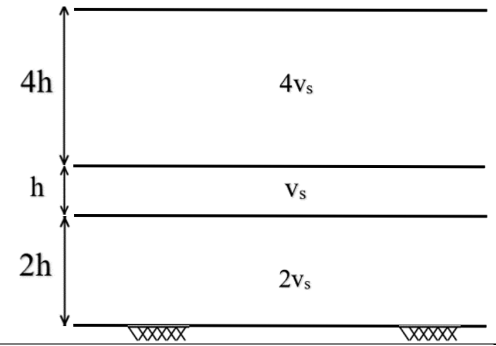
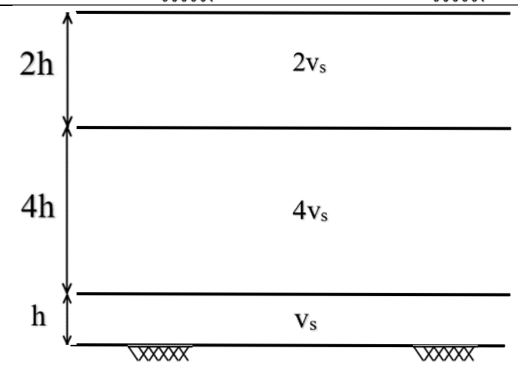
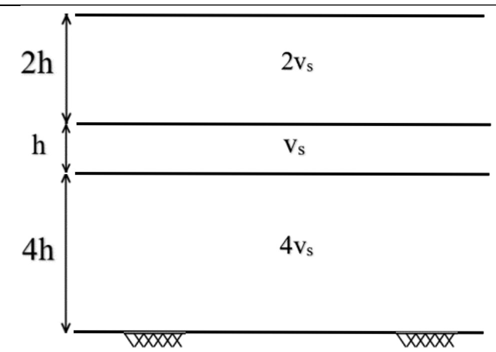
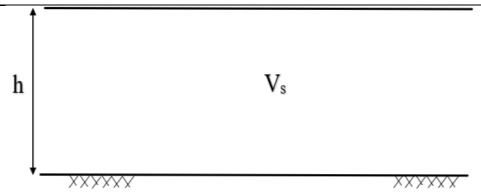
To utilize the figure (5) data, every one of the two layers ratio should be a constant value. For additional explanation, the ratio of strong layer to average layer and the ration of medium and weak layers should be a constant. As indicated by the figure (5), the natural period of the soil diminishing with increasing stiffness ratio. The results reverse if the layers are from strong to weak. For states (4-b), (4-c), (4-d) and (4-e) the natural period of the soil is same. Maximum value for periods is observed in states (4-a) and (4-f). It ought to be noticed that, the soil natural period increases with increasing the stiffness ratio (in case of (4-a)).

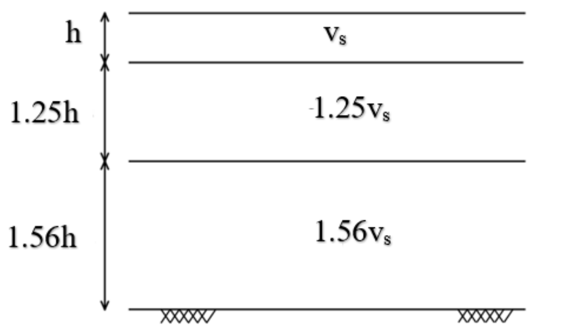
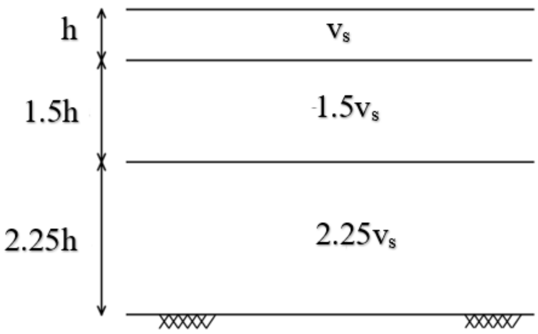
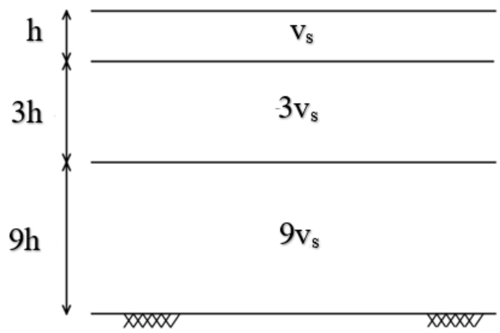
2.4. Three layers soil verification

This section is to verify the proposed method with Wang et al., (2018) [27]. The request for the layers is expected as Figure 4. Three methods of layer arrangement are considered and compared with an equivalent layer. The outcomes have been displayed in table 2.

Table 2. ground period with considering the three layer and one layer soil (proposed method and Wang et al. (2018))

Layers order	Soil profile	Soil period	Wang et al., (2018) [27] $T = \pi \sum_{i=1}^{n} \frac{h_i}{v_i}$	Error (%)
Weak, average, strong		$T = \frac{8.217h}{v_s}$	$T = \frac{9.42h}{v_s}$	14.6
Weak, strong, average		$T = \frac{2.407h}{v_s}$	$T = \frac{9.42h}{v_s}$	290

Strong, average, weak		$T=\frac{20.08h}{v_s}$	$T=\frac{9.42h}{v_s}$	53
Strong, weak, average		$T=\frac{2.246h}{v_s}$	$T=\frac{9.42h}{v_s}$	319
Average , strong, weak		$T=\frac{2.2465h}{v_s}$	$T=\frac{9.42h}{v_s}$	319
Average , weak, strong		$T=\frac{2.4073h}{v_s}$	$T=\frac{9.42h}{v_s}$	291.3
One layer		$T=\frac{4h}{v_s}$	$T=\frac{3.14h}{v_s}$	21

Weak, average, strong		$T = \frac{10.67h}{v_s}$	$T = \frac{9.42h}{v_s}$	11.7
Weak, average, strong		$T = \frac{9.1h}{v_s}$	$T = \frac{9.42h}{v_s}$	3.5
Weak, average, strong		$T = \frac{7.21h}{v_s}$	$T = \frac{9.42h}{v_s}$	30.6

the order of the layers in Wang et al. (2018) formula have been dismissed. Subsequently, the results contain critical errors. Clearly, weak soil at the least layer cause to increase the ground period. The layers level in every one of the circumstances is h; in three-layer h is separated to the three sections with consistent ratio.

2.5. Soil period base on the Cha and Gye (2007) study [28]

According to the table 3, soil profile consists of the 12 layers with specific depth and shear wave velocity.

Table3. Soil particulars for borehole DH-5 of the Cheju site [28]

Depth (m)	Soil Type	Specimen	σ'_v (kPa)	Measured $V_{s\ field}$ (m/s)	Estimated e_{field} (Eq 4)	Estimated ϕ (°)	Average ϕ by N (°)	Difference (%)
4	SP	Cheju (sand)	38.4	203	1.07	38.3	40.2	5.0
5	SP		48.0	210	1.09	38.0	35.7	6.1
6	SP		57.6	212	1.13	37.4	33.2	11.2
7	SP		67.2	223	1.09	37.9	34.4	9.2
8	SP		76.8	226	1.11	37.7	34.5	8.5
9	SP		86.4	243	1.03	39.0	33.8	13.3
10	SP		96.0	230	1.16	36.9	32.3	12.5
11	SP		105.6	213	1.30	34.5	30.7	11.0
12	SP		115.2	188	1.36	33.5	29.3	12.5
13	SP		124.9	219	1.31	34.4	32.1	6.7
14	SP		134.5	236	1.22	35.9	N/A	N/A
15	SP		144.1	216	1.36	33.5	N/A	N/A

In this study, twelve depth and shear wave velocity divide to three weak, strong, and mean distinct depth (see figure 6),

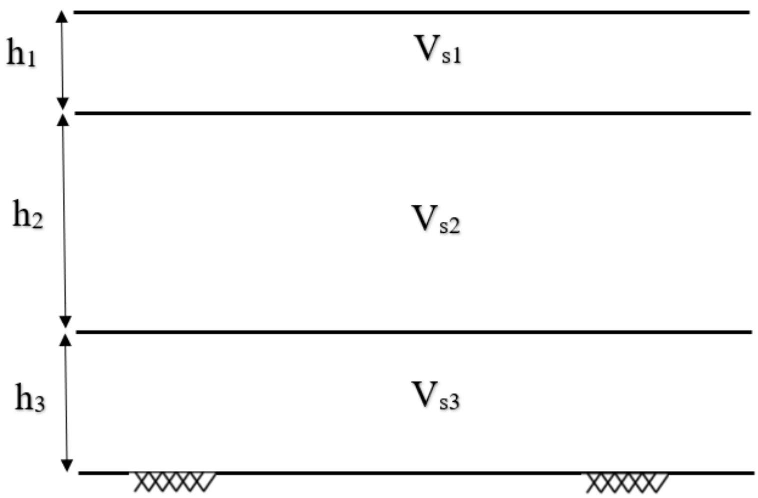


Figure 6. Schematic of equivalent layers Cha and Gye (2007) [28] study

The average shear velocities are determined as follows:

Layers depth is

$$v_{s1} = \frac{203 + 210 + 212}{3} = 208.3\ (\frac{m}{s})$$

$$v_{s2} = \frac{223 + 226 + 243 + 230 + 213}{5} = 227(\frac{m}{s})$$

$$v_{s3} = \frac{188 + 219 + 236 + 216}{4} = 214.75(\frac{m}{s})$$

And layers depth is

$h_1 = 3, h_2 = 5, h_3 = 4\ (m)$

In the following step, the soil layers particular ratio is determined

$$\frac{h_3 \times v_{s3}}{h_1 \times v_{s1}} = \frac{4 \times 214.75}{3 \times 208.3} = 1.374$$
$$\frac{h_2 \times v_{s2}}{h_3 \times v_{s3}} = \frac{5 \times 227}{4 \times 214.75} = 1.321$$

As per above relations, mean ratio approximately is

ratio=1.347

As per the figure (5-e) the coefficient of period for the ratio 1.347 is 2.4, then, at that point,

$$T = \frac{\text{coefficient} \times \mathrm{h}_1}{v_{s1}} = \frac{2.4 \times 3}{208.3} = 0.035(s)$$

Soil period around the borehole DH-5 is 0.035 second. This relation has been obtained based on the equation (36), which shows the three-layer natural ground period.

In this study the process of calculations and outputs has been performed using MATLAB software; the codes for computing the natural period of three-layer soil is given in Appendix 1.

3.Conclusion

According to the purpose of the research, which is to accurately calculate the natural period of multilayer soils, the process of solving equations was done explicitly (by vibration method) in the methodology section and the final displacement equation was calculated. The equation presented in this research is the simplest equation that has greatly shortened the time and solution process in terms of calculations. At long last, to calculate the natural period of the soil, two layers or three layers are enough to put the provided relation in the software for solving mathematical equations such as MATLAB and calculate the exact solution of the soil vibration. Categorical relations and results are presented in the form of graphs that are more understandable and simplified for engineering calculations. It tends to be presumed that when the weak layer is in the lowest layer, the natural period of the soil is significant and ground displacements is high. In other cases, the natural period of the soil is a small value and approximately is in a same range. To calculate the cases which are outside the examples in this study, is adequate to utilize equation 30 for two layers ground and equation 36 for three layers.

In this study, the presented equations (30 and 36) are obtained by the explicit analytical method. Hence, are the simplest equations in comparison with previous studies and with accurate outputs.

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Appendix 1.

MATLAB codes used in this study to calculate the natural period of three-layer soils in specific conditions is presented as follow:

```
%.........................................define parameters…………………………….………%
syms vs1 vs2 vs3 k1 k2 k3 h1 h2 h3 alfa1 alfa2 alfa3 a b c h vs k omega
%................................ layers specifications ratio…………………………….%
x=[2];
%................................ layers height and velocity…………………………….%
h1=1*h;
h2=(1*x(i,1)*x(i,1))*h;
h3=(1*x(i,1))*h;
vs1=1*vs;
vs2=(1*x(i,1)*x(i,1))*vs;
vs3=(1*x(i,1))*vs;
%................................ solving process…………………………….%
k1=omega/vs1;
k2=omega/vs2;
k3=omega/vs3;

alfa1=vs1/vs2;
alfa2=vs2/vs3;
alfa3=vs1/vs3;

a=k1*h1;
b=k2*h2;
c=k3*h3;
%................................ omega formula……………………….……….%
e=cos(a)*cos(b)*cos(c)-alfa1*sin(a)*sin(b)*cos(c)-alfa2*cos(a)*sin(b)*sin(c)-
alfa3*sin(a)*cos(b)*sin(c);
%..........................simplifying and period calculation…………….%
e_2=simplify(e);
e_3=solve(e_2,omega);

T=2*pi/e_3;
T=vpa(T);
T_org=T(1);
```